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PROBLEMS IN NAVIGATION

AND

NAUTICAL ASTRONOMY

BY

H. B. GOODWIN, M.A.

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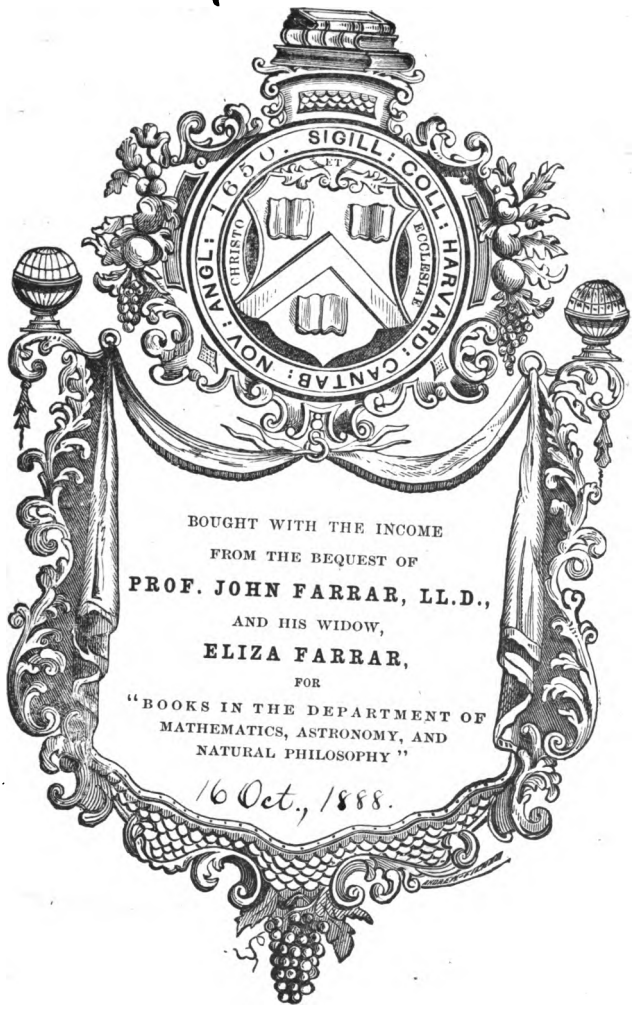

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PROBLEMS IN NAVIGATION

AND

NAUTICAL ASTRONOMY

PROPOSED IN THE EXAMINATIONS FOR RANK OF LIEUTENANT, HELD AT THE ROYAL NAVAL COLLEGE BETWEEN THE YEARS 1874 AND 1886.

WITH ANSWERS AND HINTS TO SOLUTION.

BY
H. B. GOODWIN, M.A.,

NAVAL INSTRUCTOR ROYAL NAVY.

(For some years Examiner in Navigation and Nautical Astronomy at the Royal Naval College, Greenwich.)

ILLUSTRATED WITH 83 DIAGRAMS.

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PREFACE.

"Secondly,—Those learners who are desirous of being instructed in the Art of Navigation in a scientific manner, and would chuse to see the reason of the several steps they would take to acquire it."—(Robertson.)

WHEN, more than a century ago, one John Robertson published his great work on Navigation, the reputation of which remains unimpaired to this day, he divided his probable readers into four classes, ranging from the trained mathematician who would look into his book in search of some new thing, to the simple master mariner who asked only plain rules of thumb, and would not pretend to care for the reasons upon which these rules might be based.

Robertson's classification may be accepted as tolerably correct at the present time, and as it is chiefly in the interest of the second of his four classes that this collection of problems has been made, the specification of that class (somewhat more numerous perhaps now than a hundred years ago) is therefore given literally above.

The volume consists of problems in Navigation and Nautical Astronomy taken from the "College Sheets" of the last twelve years, and is designed to supplement the groups of examples given in the various works upon these subjects used in the Naval Service.

Such examples are in a great measure limited to simple numerical illustrations of the general formulæ, which may perhaps be sufficient for the wants of beginners, but are found by the more advanced student somewhat wanting in interest and variety.

To these, however, should be added the admirable series of problems collected by the late Mr. H. W. Jeans, which are embodied in the elementary treatise on Trigonometry by the present author. A work containing Jeans' solutions of these was published some years ago by Messrs. Longmans,

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and is still on sale. It has not therefore been considered necessary to include problems already so well known in this collection, although they still appear from time to time in the Greenwich papers.

Answers to the questions are given in all cases, and whenever a problem appears to present any feature of novelty or of special difficulty, the method of solution is pointed out, explicitly enough, it is hoped, to be intelligible to students who are compelled to rely upon their own exertions. Whenever practicable, the problem is illustrated by a diagram,

the South Kensington, Board of Trade, and other examinations, in which papers are set similar in character to our own.

The questions given between August, 1879, and December, 1883, were proposed by Mr. R. E. Nelson, lately appointed Professor of Navigation in the Chinese Navy; those later than December, 1883, by Mr. J. M. Pask, R.N., the present examiner. To both of these gentlemen, as well as to the Rev. J. L. Robinson, of the Royal Naval College, my best thanks are due for their ready assistance in verifying results, and revising proof sheets.

A list of the various text-books to which reference is made from time to time is given on the next page.

H. B. GOODWIN.

ROYAL NAVAL COLLEGE,
December, 1886.

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TEXT BOOKS TO WHICH REFERENCE IS MADE.

Jeans' Navigation and Nautical Astronomy, by H. W. Jeans, Pt. I., 1870 (Longmans).

" " " Part II., 1868 (Longmans).

Merrifield & Evers' Navigation and Nautical Astronomy, 1873 (Longmans).

Raper's Practice of Navigation and Nautical Astronomy, 1874 (Potter).

Harbord's Glossary of Navigation . . 1883 (Griffin).

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PART I.

NAVIGATION.

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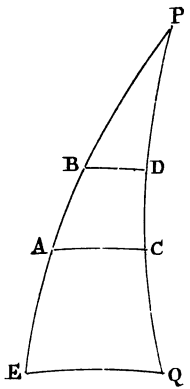
Fig. 1.



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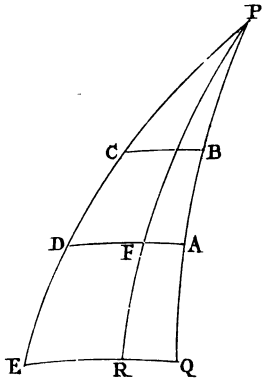

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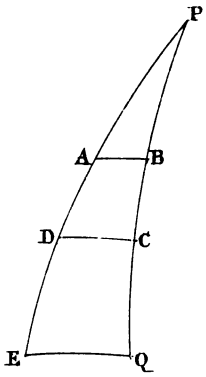

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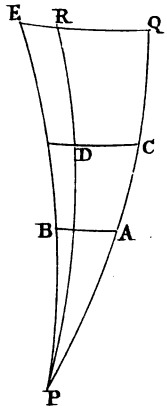

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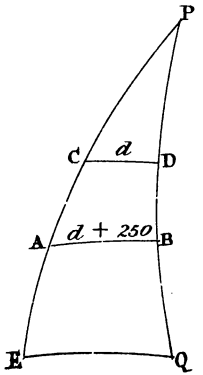

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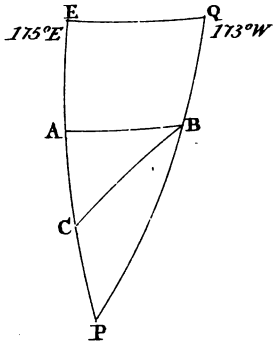

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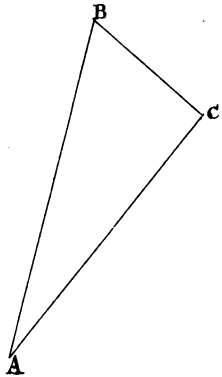

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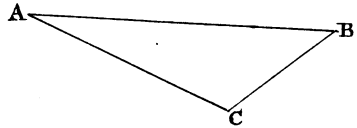

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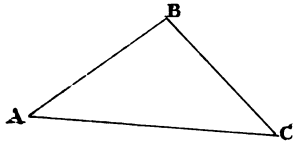

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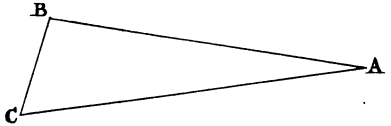

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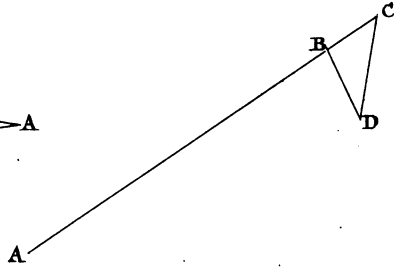

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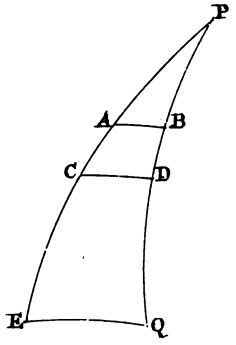

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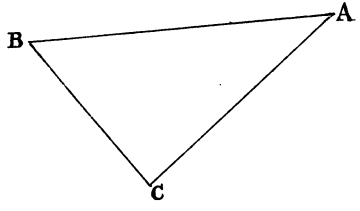

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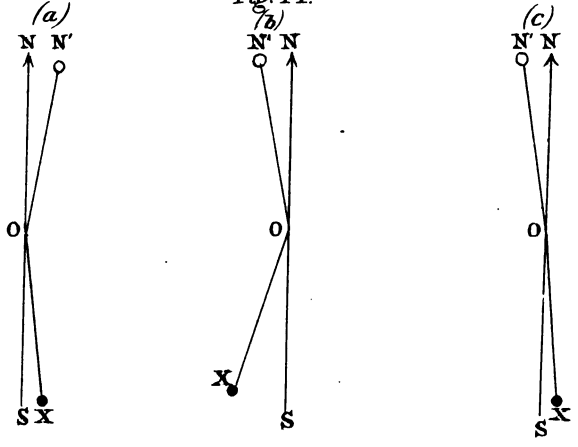

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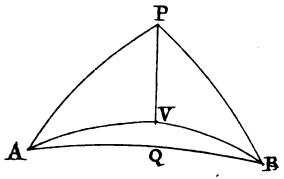

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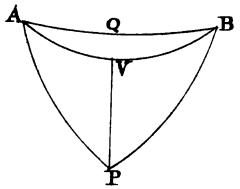

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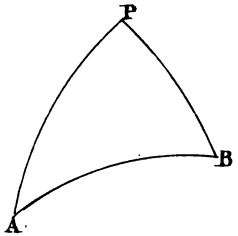

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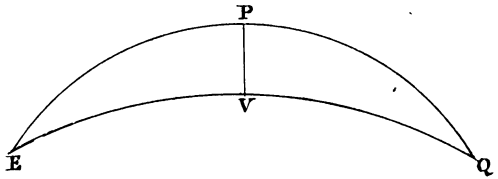

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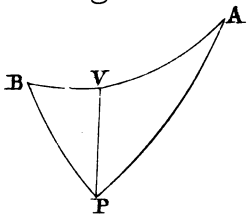

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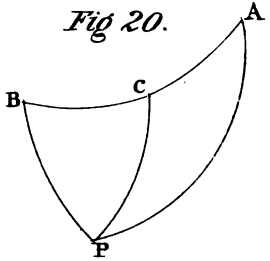

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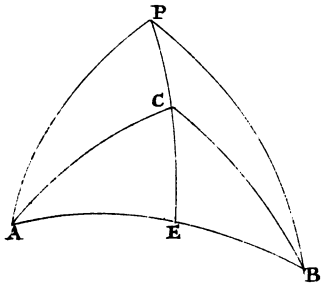

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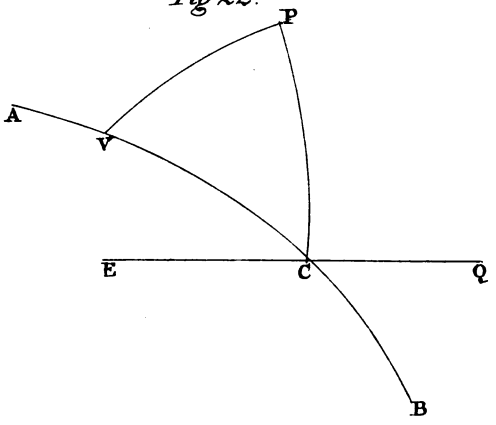

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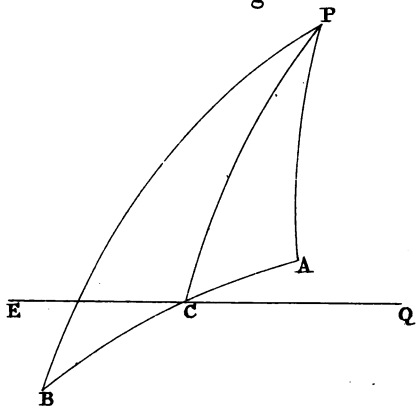

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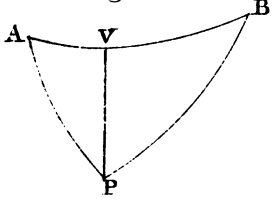

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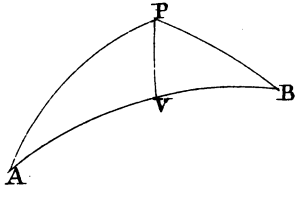

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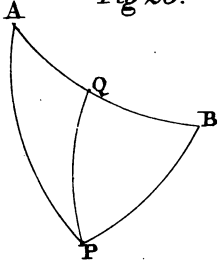

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PART I.

NAVIGATION.

I. THE SAILINGS.

1. Prove the formula made use of in parallel sailing.

Two places are in latitude 75° N., and differ in longitude by 180°. By how many miles does the length of the arc of the parallel connecting them exceed their shortest distance apart? (*May*, 1879.)

2. A ship leaves a place in latitude 41° N. for a port on the same meridian in latitude 53° N. After sailing East 295 miles, she steers due North till the latitude of her port is reached. How far is it then distant? (*October*, 1877.)

3. A vessel sailed from latitude 29° 46′ N., longitude 21° 50′ W., 350 miles due North, 350 miles due West, 350 miles due South, and 350 miles due East; required her latitude and longitude in. (*December*, 1876.)

4. A ship left a port in latitude 56° N. and sailed due East 420 miles, and then South. Having followed the meridian for some time, she altered course to West, and after sailing 520 miles reached the same longitude from which she originally started. Find the number of miles sailed on her second course. (*April*, 1876.)

5. Two ships on different parallels in south latitude were steaming W. at the same rate; the first in lat. 60° made 3′ diff. long. for every 2′ of the second. Find the latitude of the second. (*June*, 1884.)

6. A ship sailed N.N.W. from lat. 41° 50′ S., long. 18° 20′ E. until she had made good 493 miles of departure. Find her latitude and longitude in. (*November*, 1877.)

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7. A ship sailing from *A*, Lat. 46° S., Long. 76° 25′ E., to *B* (a place eastward of *A*), in Lat. 22° 21′ S., made 415 miles of departure; find the longitude of *B*. (*June*, 1877.)

8. A ship sailed W.S.W. from longitude 154° 30′ W. until she reached latitude 47° 25′ N., longitude 178° 50′ W. Required her latitude from. (*February*, 1877.)

9. A place in a certain latitude is carried by the rotation of the earth 250 miles an hour faster than another place 22° nearer the pole. Find the latitude of each. (*October*, 1878.)

10. Under what circumstances is the Middle Latitude formula of advantage, and when should its use be avoided?

In sailing 107′ N.E. by E. a ship changed her longitude 1° 43′; find the latitudes (North) from and in. (*March*, 1885.)

11. A ship in the Northern Hemisphere found the distance sailed one day to be double the difference of latitude and equal to the difference of longitude. Find the middle latitude. (*February*, 1885.)

12. A ship in north latitude sailed from A to B, making 320 miles N., 417 miles departure, and 8° 50′ difference of longitude (E.)

The longitude of A being 40° W., find the latitude and longitude of B. (*April*, 1884.)

13. A ship in south latitude, longitude 175° E., sailed due East until she reached the meridian of 173° W., and then sailed S.W.b.S. until she reached the meridian from which she started, her latitude being then 43° S. Find her latitude from. (*June*, 1876.)

14. A ship sailed from a place in North latitude N.W. by W. until she had made 97·6 miles of departure, her Meridional Difference of Latitude being 126·5. Required the latitude left and the latitude in. (*April*, 1878.)

15. A ship sails from lat. 58° 10′ N., long. 43° 30′ W., between South and East, till she arrives in long. 40° 12′ W., and her M.D. lat. is 520 miles; find the course and distance made good, and the latitude in. (*October*, 1880.)

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16. A ship sails from a place A in latitude 48° 31′ N., longitude 37° 30′ W., due South 89 miles, and then due East till she arrives at a place B in longitude 35° 32′ W. Another ship sails from A to B on a direct course. Both ships start together and arrive together. Compare their rates of sailing. (*August*, 1882.)

17. Two ships, A and B, are chasing to windward, the wind being N.E. At the end of a given time A bears from B, S. by W. $\frac{3}{4}$ W., distant 2,500 yards. Find by the Traverse Table how much B has weathered on A. (*May*, 1881.)

18. Solve the following question by means of the Traverse Table: A ship steaming E. $\frac{1}{2}$ S. 7·5 knots, found that she had made good in 3 hours, 20 miles E.S.E. Find the direction and rate of the current. (*April*, 1877.)

19. By dead reckoning a ship has sailed N.E.b.E. 29 miles in 5 hours; by cross bearings it is found that she has made good 41 miles E. $\frac{1}{2}$ S. Find by the aid of the Traverse Table the direction and rate of the current by which she was set. (*December*, 1876).

20. Find the compass course to be steered by a vessel which can steam 9 miles an hour, in order to reach a port bearing W. by S. (true) from her, distant 95$\frac{3}{4}$ miles, and the time in which she will reach it. The Variation and Deviation together are 25° W., and a current sets the ship S. by W. 2 miles an hour. (*November*, 1880.)

21. A certain port bore from a ship N.E. by E. and was 85 miles distant. After steering that course for 7 hours at a speed of 10$\frac{1}{2}$ knots, the port bore N. by E., and was 21 miles distant. Find the direction and rate of the current by which she was set. (*March*, 1879.)

22. Explain the construction of a Mercator's Chart, and investigate a formula for finding approximately the meridional parts for any given latitude.

The meridional parts for lat. 48° 51′ are 3368·39; calculate those for lat. 48° 56′. (*August*, 1882.)

23. Two ships sail due East at uniform rates of 4 and 5

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miles per hour respectively, their difference of latitude is 10°, and they change their longitude at the same rate. Find the latitude of each. (*November*, 1883.)

24. Two ships leave a port in North latitude, sailing at the same speed N. $x°$ W., N. $2x°$ W. respectively. At noon next day the difference of latitude made good by the two ships was 90′ in the one case and 15′ in the other. Find the distance sailed. (*June*, 1879.)

II. THE LOG LINE.

N.B.—In the following questions the length of the nautical mile should be taken as 6080 *feet.*

1. A ship ran a distance of 150 miles (supposed), using a 28-seconds glass and a 50-feet log-line. Determine her true distance made good. (*November*, 1878.)

2. A ship, using a 26-second glass, ran a distance of 150 miles as entered in her Log. If the true distance run was only 144 miles, find the length of the knot of the log-line used. (*April*, 1876.)

3. A ship using a 14-second glass logged 285 miles in a run of 270; what was the length of a knot on the line used? (*November*, 1884.)

4. If with a 26-second glass the length of a knot on the log-line be taken as 45 feet, find the error in an apparent run of 200 miles. (*September*, 1879.)

5. A ship in a run of 240 miles (by log) was overlogged 18 miles; find the error in length of a knot on the line, a 28-second glass having been used. (*June*, 1885.)

6. By dead reckoning, a ship has sailed 117 miles W. $\frac{1}{2}$ S. in 18 hours. By cross bearings it appears that she has made good 102 miles S.W. $\frac{1}{4}$ W. The log-line which she used, supposed to be 51 feet, was 2·5 feet too long. Find the direction and rate of the current. (*October*, 1877.)

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7. A ship steaming from 2° N., 2° W. at a rate by log of 6·8 knots, and using a 28-second glass, reached 2° S., 2° E. in 48 hours.

Find the error in length of a knot on the line. (*December*, 1884.)

8. A ship at noon on Sunday was in lat. 32° 30′ N.; her course was then shaped S.E. by E. (true), and she reached the tropics the following Thursday at 6 p.m. She averaged by log 9 knots. Find the error in length of the log-line, a 28-seconds' glass having been used. (*May*, 1884.)

III. DEVIATION OF THE COMPASS.

1. In swinging ship for deviation, the bearings taken on board were S. by E., S.S.W. ¼ W., S. ¼ W., and those taken ashore N. ¼ W., N.b.E. ¼ E., and N. ¾ W. Give diagrams, and find the amount of the deviation in each case. (*April*, 1878.)

2. In swinging ship for deviation, the following bearings were observed; find the deviation in each case:

Bearings of the Ship Compass from the shore, N. ¼ E. S.W. by W., W. ½ N.

Corresponding bearings of Shore Compass, S. ¼ E., N.E. ½ E., E. ¼ N. (*November*, 1885.)

3. Drawing a figure in each case, find the bearings of the ship's compass when the bearings of the shore compass were N. by E., W., and S. by E.; and the deviations 10° W., 8° E., and 7° W. respectively. (*February*, 1884.)

4. In swinging ship for deviation of compass, the bearings of the ship's compass were W.b.S. ½ S., W.S.W., S.W. ½ W., and the respective deviations 2¼ pts. E., 1½ pts. W., ¾ pt. E. Required the bearings of the shore compass in each case. (*March*, 1876.)

5. In swinging ship for deviation of compass, the bearings of the ship's compass were W.b.N. ¼ N., N.W. ¾ W., and W. ¼ N., the respective deviations being ¾ E., ¼ W., and 1¼ W.

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Give diagrams, and find the bearing of the shore compass in each case. (*May*, 1877.)

6. In swinging ship for deviation of compass, the bearings of the shore compass were S.E. ½ E., E.S.E., and E. ¼ N., the respective deviations being ¾ pt. W., ½ pt. E., and 1¼ pt. E. Give diagrams, and find the bearing of the ship's compass in each case. (*September*, 1875.)

IV. GREAT CIRCLE SAILING.

1. What saving will be effected by sailing on the arc of a great circle, instead of the parallel, between two places whose latitude is 45° and difference of longitude 90°? (*November*, 1884.)

2. Two places are on the same parallel of latitude, and the distance between them, measured on a great circle, is 1,800 miles. The highest latitude reached in sailing from one to the other is 63° S. Find the distance saved by following the great circle instead of the parallel of latitude connecting the two places. (*December*, 1880).

3. A ship sails on a great circle from latitude 37° 10′ N., longitude 38° 20′ W.; her first course is N. 58° E., and her last course N. 72° E. Find the latitude and longitude arrived at. (*May*, 1880.)

4. Explain what is meant by the vertex of a great circle. How can we tell, having given the first and last courses between two places upon the same side of the equator, whether the vertex will fall on the arc joining the places, or upon the arc produced?

The vertex of a great circle is 90° from the point where the arc joining the two places, produced if necessary, cuts the equator. Prove this. (*April*, 1879.)

5. *A* and *B* are two places in South latitude. The highest latitude reached by a ship sailing on a great circle between *A* and *B* is 57° 40′ S.; her first course was S.W. and her last course N. 67° W. Required the difference of longitude between *A* and *B*. (*February*, 1876.)

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6. A ship starts on a great circle from A, lat. 35° 36′ S., long. 19° 36′ W., for B, lat. 43° 27′ S., long. 70° 18′ W. On reaching the latitude of B she alters her course to West, and follows the parallel till she arrives at her port. Find the distance sailed on the great circle. (*October*, 1878.)

7. The distance on a great circle, between two places, A (lat. 40° N., long. 15° E.) and B (lat. 25° N., and to the Eastward of A) is 20°; find the latitude and longitude of a third place C, to the Northward of, and 20° distant from, both A and B. (*April*, 1880.)

8. Show that when a ship sails on a great circle from a place in North latitude to a place in South latitude, or *vice versâ*, her course on crossing the equator is equal to the complement of the latitude of the vertex.

A ship sails on a great circle from A, in lat. 18° 11′ N., to B, in South latitude, and in longitude 38° 11′ W. She crosses the equator in longitude 27° W., and her course is then W.S.W. Find the latitude of B, and the distance sailed. (*April*, 1883.)

9. The latitude of A is 52° S., of B 45° S., and A is 600 miles nearer to the vertex of the great circle joining A, B than B. Find their difference of longitude.

10. A ship sails on a great circle from A to B. The latitudes of A and B are complementary to each other, the latitude of the vertex is 60° N., and the distance from A to the vertex is double the distance from B to the vertex. Find the latitudes of A and B, and the distance sailed. (*December*, 1883.)

11. Find the latitude of the point of "Maximum Separation in Latitude" in sailing from lat. 41° 30′ S., long. 118° E., to lat. 47° S., long. 171° 50′ E. (*November*, 1878.)

12. Two ships sail from lat. 46° S., long. 105° E., to lat. 21° S., long. 48° E., the one upon the rhumb line, the other upon the great circle joining the two places. Find the latitude of the latter, when the two ships are sailing in directions parallel to one another. (*June*, 1879.)

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PART I.

NAVIGATION.

ANSWERS AND HINTS FOR SOLUTION.

I.—THE SAILINGS.

1. The shortest distance between the two places is the arc of the great circle joining them.

Since in the present case the difference of longitude is 180°, it is evident that the great circle joining the places passes through the Pole.

The length of this arc is therefore 30°, or 1800 nautical miles.

For the distance on the parallel we have (*Jeans' Naut. Ast. and Navigation*, Part II., p. 150)—

$$\text{Distance} = \text{Diff. Long.} \times \cos. \text{lat.}$$
$$= 10{,}800 \times \cos. 75° = 2795{\cdot}3 \text{ miles.}$$

Thus the distance on the parallel exceeds the shortest distance by (2795·3 − 1800) miles, or 995·3 miles.

Ans., 995·3 miles.

2. *See fig.* 1.

Let EQ be the equator; PE, PQ be meridians; A the first place of the ship; D her last place; B the port which she wishes to reach.

Let x = the distance required, l = the diff. long. EQ.

Then from the figure

$$x = l \cos. 53°, \text{ and } l = 295' \text{ sec. } 41°.$$

Substituting for l in the first equation

$$x = 295' \text{ sec. } 41° \cos. 53° = 235{\cdot}2 \text{ miles.}$$

Ans., 235·2 miles.

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3. *See fig.* 2.

Let A be the first, F the last place of the ship.

Since the northing and southing made good are equal, the latitude in is clearly 29° 46′ N.

To find the longitude in :—

From the data

$$FD = BC.$$

And RQ the diff. long. $= EQ - ER = BC \text{ sec. } 35° 36' - BC \text{ sec. } 29° 46'.$

$$= 430'5 - 403'2 = 27'3$$

Thus the longitude in is 21° 50′ W. + 27′·3 W. = 22° 17′ W.

Ans., Lat. in 29° 46′ N. } Long. in 22° 17′ W. }

4. *See fig.* 3.

Let $x°$ = lat. in, l = diff. long.

Then from the figure

$$AB = 420' = l \text{ cos. } 56°.$$

$$CD = 520' = l \text{ cos. } x°.$$

Thus, dividing one equation by the other,

$$\text{Cos. } x° = \frac{520}{420} \text{ cos. } 56°, \text{ whence } x = 46° 11'.$$

And BC the distance sailed on the second course $= 56° - 46° 11'.$

$$= 9° 49'.$$

$$= 589 \text{ miles.}$$

Ans., 589 miles.

5. *See fig.* 4.

Let AB be the track of the ship in lat. 60°, for a time t.

Let CD be the track of the second ship in lat. $x°$ for the same time.

Then if EQ, the diff. long. made good by the first ship $= 3l$ QR the diff. long. of the second ship $= 2l$. Let AB or CD $= d$.

From the figure

$$EQ \text{ or } 3l = d \text{ sec. } 60° = 2d.$$

and $$QR \text{ or } 2l = d \text{ sec. } x°.$$

Therefore $\frac{3l}{2l} = \frac{2d}{d \text{ sec. } x°}$, whence sec. $x° = \frac{4}{3}$, and $x° = 41° 24' 30''$.

Ans., 41° 24′ 30″.

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6. By formulæ (*Jeans' Naut. Ast. and Navigation*, Part II., pp. 148, 149),

True diff. lat. = Dep. × cot. course.
= 493′ cot. 22° 30′.
= 1190′ = 19° 50′.

Thus lat. in = 41° 50′ S. − 19° 50′ N. = 22° S.

Again, by formula (p. 150),

D. long. = MD. Lat. × tan course.
= 1414′·6 tan 22° 30′ = 585′·9.

Thus long. in = 18° 20′ E. − 9° 46′ W. = 8° 34′ E.

Ans., Lat. in 22° S.
Long. in 8° 34′ E.

7. Lat. from 46° 0′ S. MP. 3115·55
„ in 22° 21′ S. „ 1376·36
23° 39′ N. MD. Lat. 1739·19

Therefore TD Lat. = 1419′

By formula.

$$\frac{\text{D Long.}}{\text{MD Lat.}} = \text{tan course} = \frac{\text{Dep.}}{\text{TD Lat.}}$$

Therefore,

$$\text{D Long.} = \frac{\text{Dep.} \times \text{MD Lat.}}{\text{TD Lat.}} = 508'{\cdot}6 = 8^\circ\ 29'.$$

And Long. in = 76° 25′ E. + 8° 29′ E. = 84° 54′ E.

Ans., 84° 54′ E.

8. Long. in 178° 50′ W.
Long. from 154 30 W.
24° 20′ W.
60
Diff. Long. 1460′ W.

By formula.

MD Lat. = Diff. Long. × cot. course = 1460′ cot. 67° 30′.

3·164353
9·617224
2·781577
604·7 S.

MP for 47° 25′ N. . . 3239·5
MD Lat. . . . . 604·7
MP for Lat. from . . 3844·2
Lat. from 53° 48′.

Ans., 53° 48′ N.

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9. *See fig.* 5.

Let CD be the arc described in an hour by a point in lat. $x°$.

Let AB be the arc described in an hour by a point in lat. $(x° - 22°)$.

Then EQ, the arc described in the same time by a point on the equator $= 15° = 900$ miles.

Thus we have—

$$d + 250 = 900 \cos.\ (x° - 22°)$$
$$d = 900 \cos.\ x°.$$

Subtracting the second equation from the first, we obtain—

$$250 = 900\ \{\cos.\ (x° - 22°) - \cos.\ x°\}.$$

Therefore $2 \sin.\ 11° \sin.\ (x° - 11°) = \frac{5}{18}$.

$$\text{and } \sin.\ (x° - 11°) = \frac{5}{36} \operatorname{cosec.}\ 11°$$

$$\text{Whence } x° = 57°\ 42'\ 30''$$

Ans., 35° 42′ 30″ and 57° 42′ 30″.

10. *See Raper's Practice of Navigation. Art.* 332.

By formula,

$$\text{Departure} = \text{distance} \times \sin.\ \text{course}.$$
$$= 107' \times \sin.\ 56°\ 15'.$$
$$= 89'.$$

Again, by formula,

$$\text{Cos. Mid. Lat.} = \frac{\text{departure}}{\text{diff. long.}} = \frac{89.}{103.}$$

And Mid. Lat. $= 30°\ 15'\ 30''$ N.

We have now to find the Diff. Lat.

By formula,

$$\text{Diff. Lat.} = \text{distance} \times \cos.\ \text{course}.$$
$$= 59'{\cdot}44.$$
$$= 59'.26''.$$

Then—

Lat. from $= 30°\ 15'\ 30''$ N. $- 29'\ 43'' = 29°\ 45'\ 47''$ N.

Lat. in $= 30°\ 15'\ 30''$ N. $+ 29'\ 43'' = 30°\ 45'\ 13''$ N.

Ans., Lat. from 29° 46′ N. } Lat. in 30° 45′ N. }

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11. Let l = the diff. lat.

Then, from the data, $2l$ = diff. long. = distance.

By formula—

$$\text{Cos. course} = \frac{\text{diff. lat.}}{\text{distance}} = \frac{l}{2l} = \frac{1}{2} = \text{cos. } 60°.$$

Again, by formula—

$$\text{Departure} = \text{distance} \times \text{sin. course} = 2l \frac{\sqrt{3}}{2} = \sqrt{3}l.$$

$$\text{And—cos. Mid. Lat.} = \frac{\text{dep.}}{\text{diff. long.}} = \frac{\sqrt{3}l}{2l} = \frac{\sqrt{3}}{2} = \text{cos. } 30°.$$

Ans. 30° N.

12. By formula—

$$\text{Cos. mid. lat.} = \frac{\text{departure}}{\text{diff. long.}} = \frac{417}{530} = \text{cos. } 38° 6' 45''.$$

Then Lat. B = 38° 6′ 45″ N. + 2° 40′ = 40° 46′ 45″ N.

Long. B = 40° W. − 8° 50′ = 31° 10′ W.

Ans.—Lat. B, 40° 47′ N.; Long. B, 31° 10′ W.

13. *See fig.* 6.

By formula—

MD Lat. = D Long. × cot. course.
= 720 × cot. 33° 45′.
= 1077·6

MP for 43° . . .	2863·10
MD Lat.	1077·60
MP for lat. from . .	1785·50

∴ Lat. from . . . = 28° 30′ S.

Ans. 28° 30′ S.

14. By formula—

Diff. long. = MD Lat. × tan course.
= 126′·5 × tan 56° 15′.
= 189′·3.

$$\text{And cos. mid. lat.} = \frac{\text{dep.}}{\text{diff. long.}} = \frac{97·6}{189·3} = \text{cos. } 58° 57' 45''.$$

Thus, MP for lat. from = MP for 58° 58′ − 63·25.

MP for lat. in = MP for 58° 58′ + 63·25.

Ans. { Lat. from = 58° 25′ N.
Lat. in = 59° 30′ N.

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15. Diff. long. = 3° 18′ E.

By formula—

$$\text{Tan course} = \frac{\text{diff. long.}}{\text{MD lat.}} = \frac{198}{520} = \tan 20° 50′ 45″.$$

MP for lat. 58° 10′ N. . . .	4313·21
MD lat.	520·00
MP for lat. in	3793·21
Lat. in	53° 18′ N.

And distance = TD lat. × sec. course.

= 312′·5.

Ans. S. 20° 51′ E. 313 miles; Lat. in 53° 18′ N.

16. The distance run by the first ship is 89 miles south, and then 80·4 miles east; the total being 169·4 miles.

The distance run by the second ship will be found to be 119·2 miles.

Thus the rates of speed are nearly as 1·421 to 1.

17. *See fig.* 7.

On AB describe a semicircle, and from A draw AC in a north-easterly direction, meeting the circumference in C.

Then AC is the distance gained to windward by B upon A; and AC = AB cos. BAC = 2500 yds. cos. 2¼ pts.

Comparing this with the formula—

TD Lat. = distance × cos. course—

we see that if we look out 250 as a distance, and 2¼ points as a course, the distance gained to windward will be ten times the quantity given under Diff. Lat.

This quantity will be found to be 226.

Thus the distance which B has weathered on A is 2260 yds.

Ans. 2260 yards.

18. *See fig.* 8.

Let AB be the course steered; AC the course made good; then BC represents the effect of the current.

			S.	E.
Course steered,	S. 7½ pts. E.	22′· 5	2·2	22·4.
Course made good,	S. 6 pts. E.	20′· 0	7·7	18·5.

Comparing these results, we obtain the southing due to current 5′·5; the westing due to current, 3′·9. And the set and drift of current will be found to be S. 35° W., 7 miles nearly.

Ans. S. 35° W., 2·33 knots nearly.

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19. *See fig.* 9.

Ans. S. 40° E., 5·2 knots.

20. *See fig.* 10.

First, to find the *true* course to be steered by the ship, allowance being made for the current:—

Let AB represent the course as steered, *i.e.*, through the water; BC the effect of the current.

Then will AC represent the course made good.

In the triangle ABC, we have AB = 9, BC = 2, and the angle ACB = 67° 30′.

Then—

$$\text{Sin. BAC} = \frac{2}{9} \text{ sin. } 67° 30' = \text{sin. } 11° 50' 45''.$$

Whence the proper course to be steered is N. 89° 24′ W., which corrected for compass error becomes N. 64° 24′ W.

And AC, which represents the speed of the ship *over the ground*, will be found to be 9·574, so that the time required will be 10 hours nearly.

Ans. N. 64° 24′ W., in 10 hours nearly.

21. *See fig.* 11.

Let A be the first position of the ship, C the port, so that AC = 85 miles. Let AB be the track of the ship *through the water*.

If from C we draw CD, S. b. W. 21 miles, D will be the actual position of the ship after 7 hours' sailing.

Thus BD is the set and drift of current.

Then we have in the triangle BCD, BC = 11·5, CD = 21, and the angle BCD = 45°.

Solving the triangle, we obtain B = 102° 43′, and BD = 15·22 miles.

Thus the set of the current is S. 21° 2′ E., and the drift 2·17 knots.

Ans. S. 21° 2′ E., 2·17 knots.

22. *See Jeans' Naut. Astronomy and Navigation*, Part II., p. 146.

MP for 48° 56′ = MP for 48° 51′ + sec. 48° 51′ + sec. 48° 52′ + sec. 48° 53′ + sec. 48° 54′ + sec. 48° 55′.

Ans. 3375·99.

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23. *See fig.* 12.

Let $l°$ and $l°+10°$ be the required latitudes.

Then $\frac{AB}{CD}=\frac{4}{5}=\frac{EQ\cos.(l°+10°)}{EQ\cos. l°}$

$$\therefore \frac{\cos. l°-\cos.(l°+10°)}{\cos. l°+\cos.(l°+10°)}=\frac{5-4}{5+4}=\frac{1}{9}$$

$$\therefore \frac{2\sin.(l°+5°)\sin. 5°}{2\cos.(l°+5°)\cos. 5°}=\frac{1}{9}$$

$$\therefore \tan.(l°+5°)=\frac{1}{9}\cot. 5°$$

Whence $l°+5°=51°\ 47'$, and $l°=46°\ 47'$.

Ans. 46° 47′, and 56° 47′.

24. Let d = the distance sailed.

By formula—True diff. lat. = distance × cos. course.

Therefore $90=d\cos. x$.

$$15=d\cos. 2x.$$

$$\therefore \text{Cos. } x=6\cos. 2x=12\cos.^2 x-6.$$

$$\therefore \text{Cos.}^2 x-\frac{1}{12}\cos. x=\frac{1}{2}.$$

Whence cos. x is found to be $\frac{3}{4}$.

And since—

$$d=90\text{ sec. } x.$$

$$\therefore d=90\times\frac{4}{3}=120'.$$

Ans. 120 miles.

II.—The Log Line.

1. *See Jeans' Nautical Astronomy and Navigation,* Part I., p. 22.

First to find the proper length of line for a 28 seconds' glass.

If x = this length, we have the proportion—

$$x \text{ ft. : } 6080 \text{ s. : : } 28 \text{ s. : } 3600.$$

Whence $x=47{\cdot}3$ feet.

To find y, the true distance sailed, we have the proportion—

$$y \text{ ft. : } 150 \text{ miles : : } 50 \text{ ft. : } 47{\cdot}3.$$

Whence y = 158·6 miles.

Ans. 158·6 miles.

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2. *Ans.* The knot should have been 43·9 feet; its actual length was 42·1 feet.

3. *Ans.* The knot should have been 23·6 feet; its actual length was 22·3 feet.

4. *Ans.* 4·9 miles too short.

5. *Ans.* 44 feet.

6. *See fig.* 13.

The distance logged would be too small, and the actual distance *through the water* is given by $\frac{53{\cdot}5}{51}117$ miles, or 122·7 miles.

Then in the triangle ABC we have AB = 122·7 miles, AC = 102 miles, and A = $3\frac{1}{4}$ pts., or 36° 33′ 45″.

Solving this triangle we obtain B = 56° 8′, whence the set of the current is S. 39° 29′ E.

And BC is 73·2 miles, so that the rate of the current is 4·07 knots.

Ans. S. 39° 29′ E., 4·07 knots.

7. First to find the distance actually made good. Since at the equator the departure is equal to the difference of longitude, we have departure = 240′ = diff. lat.

Thus the course is S.E., and distance = 240′ sec. 45° = 339′·4.

And the distance deduced from the Log is 48 × 6·8 miles, that is, 326·4 miles, so that the knot on the line was too long.

As in question 1 we may obtain the proper length of the knot for a 28 seconds' glass, 47·3 feet.

And if x be the actual length used, we have

$$x = \frac{339{\cdot}4}{326{\cdot}4}\ 47{\cdot}3 \text{ ft. or } 49{\cdot}2 \text{ ft.}$$

Ans. The knot was 1·9 feet too long.

8. First to find the distance actually made good.

We have lat. from 32° 30′ N., lat. in 23° 28′ N., so that true diff. lat. = 542′ S.

And the course is 5 points, or S. 56° 15′ E.

Thus, by formula,

Distance = true diff. lat. × sec. course—

= 542′ × sec. 56° 15′ = 975·6 miles.

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And her supposed distance was 102 × 9 miles = 918 miles, so that the length of the knot was evidently too long.

As before, the proper length may be found to be 47·3 feet, and if x be the actual length,

$$x = \frac{975{\cdot}6}{918}\ 47{\cdot}3 = 50{\cdot}3.$$

Ans. The knot was 3 feet too long.

III.—Deviation of the Compass.

1. *See fig.* 14.

See Harbord's Glossary of Navigation, p. 377.

The deviation of compass is obtained by comparing the observed *compass bearing* of some object with its *correct magnetic* bearing.

In the method of reciprocal bearings, the object of which the bearings are compared is the compass fixed ashore.

The correct magnetic bearing of this object is obtained by reversing the bearing of the ship's compass that has been observed ashore by means of the compass there stationed, the latter being unaffected by deviation.

Thus, arranging the three sets of observations under heads (*a*), (*b*), (*c*), we have in the present instance—

Correct magnetic bearing—

$$\overset{(a)}{\text{N. } 15\tfrac{3}{4} \text{ pts. E.}} \quad \overset{(b)}{\text{N. } 14\tfrac{3}{4} \text{ pts. W.}} \quad \overset{(c)}{\text{N. } 15\tfrac{1}{4} \text{ pts. E.}}$$

Compass bearing—

$$\text{N. } 15 \text{ pts. E.} \quad \text{N. } 13\tfrac{3}{4} \text{ pts. W.} \quad \text{N. } 16\tfrac{1}{4} \text{ pts. E.}$$

Let NOS be the magnetic meridian. Since the correct magnetic bearing under (*a*) is N. $15\frac{3}{4}$ pts. E., the line of bearing of X, the shore compass, is $15\frac{3}{4}$ points to the right of ON.

Thus we can lay off the position of X.

But since the compass bearing of X is N. 15 pts. E., X lies 15 points to the right of the compass needle, or the direction of the needle lies 15 points to the left of X.

Measuring back through 15 points, therefore, we arrive at ON′, the direction of the needle.

And NON′, the deviation, $= \text{XON} - \text{XON}'$.

$= 15\frac{3}{4}$ pts. $- 15$ pts.

$= \frac{3}{4}$ pts. E.

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Proceeding in the same manner with the observations under (*b*) and (*c*), we shall find that the deviation is in each case 1 pt. W.

Ans. $\frac{3}{4}$ pt. E., 1 pt. W., 1 pt. W.

2. *Ans.* $\frac{1}{2}$ pt. E., $\frac{1}{2}$ pt. E., $\frac{3}{4}$ pt. E.

3. *Ans.* S. 1° 15′ W., S. 82° E., N. 18° 15′ W.

4. *Ans.* N.E. $\frac{1}{4}$ E., E. $\frac{1}{2}$ N., N.E. $\frac{1}{4}$ N.

5. *Ans.* E. $\frac{1}{2}$ S., S.E. $\frac{1}{2}$ E., E.b.S. $\frac{1}{2}$ S.

6. *Ans.* N.W.b.W. $\frac{1}{4}$ W., N.W.b.W. $\frac{1}{2}$ W., W.b.N.

IV. Great Circle Sailing.

1. *See fig.* 15.

Let PA, PB, be the meridians passing through the two places, and let AB be the arc of the great circle joining them.

From P drop the perpendicular PV upon AB.

Since PA = PB, PV bisects the arc AB, so that AB = 2 AV.

To find AV, we have in the right-angled triangle APV,

$$\text{Sin. AV} = \text{sin. AP sin. APV}$$
$$= \text{sin. } 45°. \quad \text{sin. } 45° = \tfrac{1}{2}.$$

Thus AV = 30°, and AB = 60°, or 3,600 miles.

To find the distance on the parallel we have—

$$\text{Arc AQB} = 90° \times \text{cos. } 45° = 5400' \text{ cos. } 45° = 3818'4.$$

Thus the saving effected is (3818·4—3600) miles = 218·4 miles.

Ans. 218·4 miles.

2. *See fig.* 16.

Let A, B be the places, PA, PB the meridians passing through them, PV the perpendicular let fall from the pole upon AB, and AQB the arc of the parallel connecting A, B.

Let x = the length of the arc AQB.

Then by formula—

$$x = \text{diff. long.} \times \text{cos. lat.}$$

Thus to obtain x we must first determine the latitude, which is the complement of PA or PB, and the difference of longitude, which is represented by the angle APB, or twice the angle APV.

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In the triangle APV, we have given PV, the complement of the latitude of the vertex, 27°, AV, which is one half the length of the arc of the great circle joining A, B, that is, one half of 30°, or 15°; and the angle V is a right angle.

It is required to find PA, and APV.

Then—

Cos. PA = cos. PV. cos. AV = cos. 27° cos. 15°.

And—

Sin. PV = tan. AV. cot. APV.

So that—

Cot. APV = sin. PV. cot. AV = sin. 27° cot. 15°.

From these equations PA is found to be 30° 36′ 45″, and the latitude is 59° 23′ 15″; while the angle APV is 30° 33′, so that the difference of longitude, which is twice APV, is 61° 6′.

Then—

Cos. PA = cos. PV. cos. AV = cos. 27° cos. 15°.

And—

Sin PV = tan. AV. cot. APV,

So that—

Cot. APV = sin. PV. cot. AV = sin. 27° cot. 15°.

Thus—

Distance on parallel = 3666′ cos. 59° 23′ 15″, = 1867 miles. And the saving effected is (1867−1800) miles, or 67 miles.

Ans. 67 miles.

3. *See fig.* 17.

Let A be the place left, B the place arrived at.

Then PA, the colatitude of A = 52° 50′, the angle PAB, the first course = 58°, and the angle PBA, the supplement of the last course = 108°.

We have to find PB, the colatitude of B, and APB, the difference of longitude between A and B.

In the triangle APB,

Sin PB = sin. AP sin. PAB cosec. ABP.

= sin. 52° 50′ sin. 58° cosec. 108°.

Whence PB is found to be 45° 17′, and the latitude of B is consequently 44° 43′ N.

To find the angle ABP, we have the formula—

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$$\text{Tan.}\frac{A+B}{2}=\frac{\cos\frac{PA-PB}{2}}{\cos\frac{PA+PB}{2}}\cot.\frac{P}{2},$$

$$\text{or cot.}\frac{P}{2}=\cos.\frac{PA+PB}{2}\ \sec.\frac{PA-PB}{2}\ \tan.\frac{A+B}{2}$$

Whence P is found to be 21° 11′.

Thus the longitude of B = (38° 20′—21° 11′) W. = 17° 9′ W.

Ans. Lat. 44° 43′ N., long. 17° 9′ W.

4. *See Raper's Practice of Navigation, Art.* 343.

To shew that the vertex of a great circle is 90° from the point of its intersection with the equator.

See fig. 18.

Let EVQ be one half of a great circle, which cuts the equator in E, Q.

Let P be the pole, and EPQ the meridian passing through E, Q.

Let V be the vertex of the circle EVQ, so that the angles PVE, PVQ are each a right angle.

Then in the right-angled triangles PVE, PVQ, PE = PQ, each being a quadrant, and PV is common.

Therefore EV = VQ.

And since two great circles which intersect bisect each other, EVQ = 180°.

Therefore EV and VQ each = 90°.

5. *See fig.* 19.

In the right-angled triangles AVP, BVP, we have given the angle A = 45°, the angle B = 67°, and the side PV = 32° 20′. We have to find the angles APV and BPV, which together make up the difference of longitude.

In the triangle APV,

Cos. A = cos. PV sin. APV,

or sin. APV = cos. 45° sec. 32° 20′.

Similarly in the triangle BPV,

Sin. BPV = cos. 67° sec. 32° 20′.

Whence APV = 56° 48′ 30″, BPV = 27° 32′ 45″, and the difference of longitude is 84° 21′ 15″.

Ans., 84° 21′

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6. *See fig.* 20.

Let C be the point of intersection of the great circle drawn through A, B, with the parallel passing through B.

It is required to find the length of AC.

In the triangle APB, having given PA, PB, and the included angle APB, we may find the angle A.

This angle will be found to be 62° 34′ 15″

In the triangle ACP, having given PA, PC, and A, we may find the angle C.

This angle is 96° 13′ 30″.

Then in the same triangle, having given two sides, and the angles opposite to those sides, we may find the third side by the formula

$$\text{Tan } \frac{c}{2} = \frac{\text{Cos. } \frac{A+B}{2}}{\text{Cos. } \frac{A-B}{2}} \text{ Tan } \frac{a+b}{2},$$

And the third side will be found to be 26° 14′, or 1574 miles.

Ans. 1574 miles.

7. *See fig.* 21.

Join PC, and from C drop a perpendicular, CE, upon AB.

Then CE bisects AB.

In the triangle PAB, PA, PB, AB all being known, we may determine the angle PAB.

The angle PAB will be found to be 133° 49′.

In the triangle ACE, we have E, a right angle, AE = 10°, and AC = 20°.

Thus cos. CAE = tan 10° cot. 20°,

Whence CAE = 61° 1′ 30″.

Then the angle PAC = PAB − CAE = 72° 47′ 30″.

In the triangle PAC we now have PA, AC, and the angle PAC, from which data we find that PC = 47° 2′ 12″, and the latitude of C = 42° 57′ 48″.

To find the angle APC, which is the difference of longitude between A and C, we have the three sides of the triangle APC.

The angle APC is found to be 26° 31′, and the longitude of C is consequently 41° 31′ E.

Ans. Lat. 42° 58′ N., long. 41° 31′ E.

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8. *See fig.* 22.

Let AB be the arc of a great circle, cutting EQ, the equator, at C.

Let P be the pole. Join PC, and let fall PV perpendicular to AB.

Then in the triangle PVC, the angle PCV represents the course on crossing the equator.

And since V is the vertex, VC = 90°.

And PC = 90°.

Therefore C is the pole of the great circle passing through P, V.

Therefore PV measures the angle PCV.

See fig. 23.

Let P be the pole, C the point where the arc AB meets EQ the equator.

In the triangle PBC, PC = 90°, BPC = 38° 11′ − 27° = 11° 11′, and PCB = 90° + 22° 30′ = 112° 30′.

Then—

Sin. BPC = tan. BCP, cot. BP,

or cot. BP = sin. 11° 11′, cot. 112° 30′.

Whence BP = 94° 35′ 30″, and the lat. of B is 4° 35′ 30″ S.

And the distance sailed = AC + BC. To find AC, BC we have—

Sin. AC = cos. 71° 49′ sec. 67° 30′.

Cot. BC = sin. 112° 30′ cot. 11° 11′.

Whence AC = 54° 38′, BC = 12° 5′, so that the distance = 66° 43′ or 4003 miles.

Ans., Lat. 4° 35′ 30″ S.; distance 4003 miles.

9. *See fig.* 24.

From P let fall a perpendicular PV upon AB.

Let AV = $d°$. Then BV = 10° + $d°$.

From the right-angled triangles AVP, BVP,

cos. 38° = cos. PV cos. $d°$;

cos. 45° = cos. PV cos. $(d° + 10°)$.

$$\therefore \frac{\cos. d°}{\cos. (10° + d°)} = \frac{\cos. 38°}{\cos. 45°}$$

$$\therefore \frac{\cos. d° - \cos. (10° + d°)}{\cos. d° + \cos. (10° + d°)} = \frac{\cos. 38° - \cos. 45°.}{\cos. 38° + \cos. 45°.}$$

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$$\therefore \frac{2 \sin. (d^\circ + 5^\circ) \sin. 5^\circ}{2 \cos. (d^\circ + 5^\circ) \cos. 5^\circ} = \frac{2 \sin. 41^\circ 30' \sin. 3^\circ 30'}{2 \cos. 41^\circ 30' \cos. 3^\circ 30'}$$

$$\therefore \tan. (d^\circ + 5^\circ) = \tan 41^\circ 30' \tan. 3^\circ 30' \cot. 5^\circ$$

Whence $(d^\circ + 5^\circ) = 31^\circ\ 44'\ 15''$, and AB, which is equal to $(2d + 10^\circ) = 63^\circ\ 28'\ 30''$.

Having now in the triangle ABP the three sides AP, BP, and AB, we may determine the angle APB, the difference of longitude, which will be found to be $104^\circ\ 44'$.

Ans., $104^\circ\ 44'$.

10. *See fig.* **25.**

Let the latitude of A $= l$, then PA $= 90^\circ - l$, and PB $= l$.

Let AV $= 2d$, so that BV $= d$.

Then in the triangles APV, BPV,

$$\sin. l = \cos. 2d \cos. 30^\circ;$$

$$\cos. l = \cos. d \cos. 30^\circ$$

Squaring each equation, and adding, we have—

$$\tfrac{3}{4} (\cos.^2 2d + \cos.^2 d) = 1,$$

$$\text{and } \cos.^2 2d + \cos.^2 d = \tfrac{4}{3}.$$

Substituting for cos. $2d$ in terms of cos. d, we have—

$$4 \cos.^4 d - 4 \cos.^2 d + 1 + \cos.^2 d = \tfrac{4}{3}, \text{ or}$$

$$4 \cos.^4 d - 3 \cos.^2 d = \tfrac{1}{3}.$$

If this equation be solved we shall obtain $\cos.^2 d = \cdot 848$ nearly, and $d = 22^\circ\ 57'$, so that $3d = 68^\circ\ 51'$.

Using the value of d so found in the equation,

$$\text{Sin. } l = \cos. 2d \cos. 30^\circ.$$

And it will be found that $l = 37^\circ\ 4'$ nearly.

Ans., lat. A $= 37^\circ\ 4'$, lat. B $= 52^\circ\ 56'$, distance 5131 miles.

NOTE.—In the construction of the diagram we have assumed that the vertex lies between A and B. If, however, AV be bisected in B′, a second point will be obtained which satisfies the conditions of the problem. In this case the distance sailed $= d$, or 1377 miles.

11. *See Raper's Practice of Navigation. Art.* 345.

When a ship sails upon a great circle from one place to another in the same hemisphere, the course sailed will for some time be nearer to the meridian than the direction of the rhumb-line, but the angle between the great circle and the

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meridians will gradually increase. At the vertex this angle amounts to 90°.

If, therefore, we imagine two ships to be sailing uniformly, one along the arc of a great circle, the other along the rhumb-line joining two points, and if the two ships be supposed to alter their longitude at the same rate, so that they are at any moment upon the same meridian, the difference of latitude between them will be increased so long as the angle which the great circle makes with the meridians is *less* than the course upon the rhumb-line.

When the angle made by the great circle is equal to the course upon the rhumb-line, so that the ships are sailing in parallel directions, a maximum value for the difference of latitude is attained, and the point where this occurs is called, therefore, the "point of maximum separation in latitude."

See fig. 26.

The course upon the rhumb-line will be found to be S. 81° 52′ 30″ E.

Let then Q be the point upon the arc AB, when the ship is sailing S. 81° 52′ 30″ E., so that the angle AQP = 98° 7′ 30″.

In the triangle APB, having two sides PA, PB, and the included angle APB, we may find the angle A.

This will be found to be 62° 57′ 15″.

And in the triangle AQP,

Sin. PQ = sin. 48° 30′. sin. 62° 57′ 15″. cosec. 98° 7′ 30″, whence PQ = 42° 21′ 45″, and the latitude is 47° 38′ 15″ S.

Ans.; Lat. 47° 38′ S.

12. *Ans.*, Lat. 38° 51′ S.

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PART II.

NAUTICAL ASTRONOMY.

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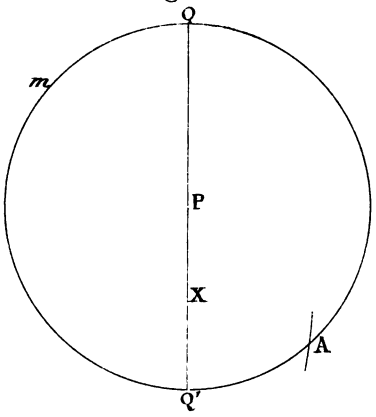
Fig 1



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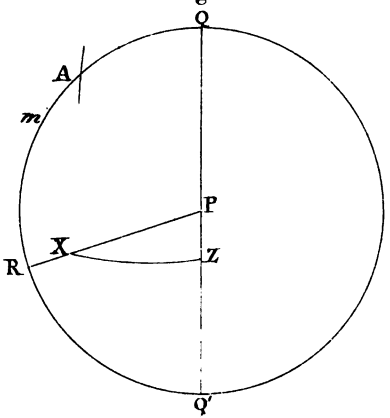

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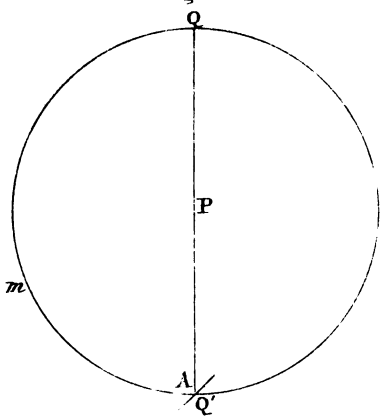

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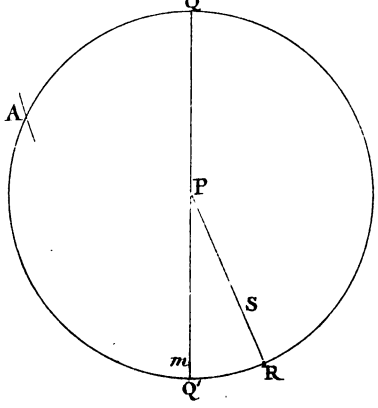

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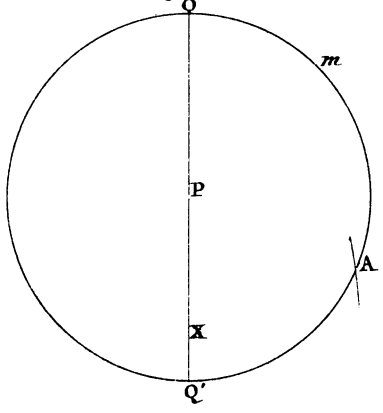

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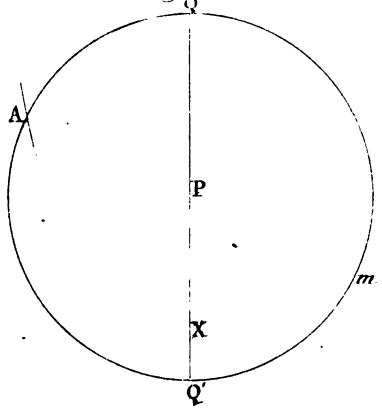

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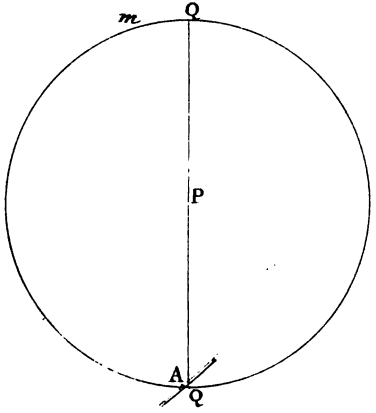
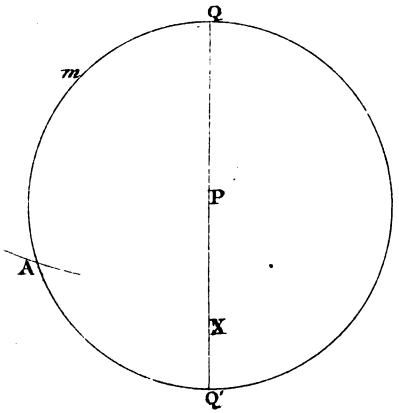


Fig 8



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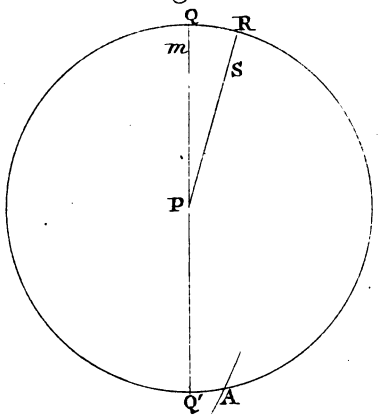

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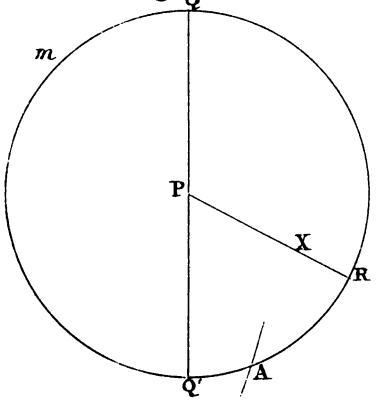

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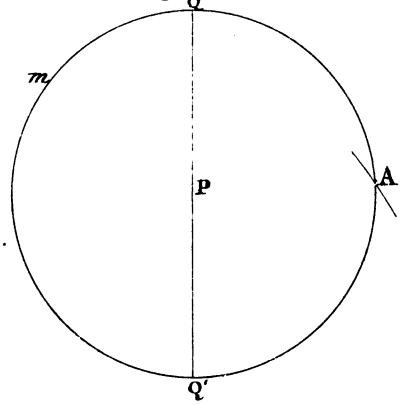

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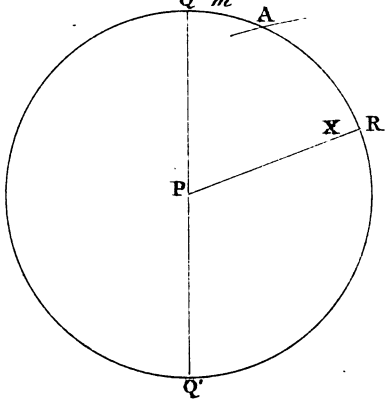

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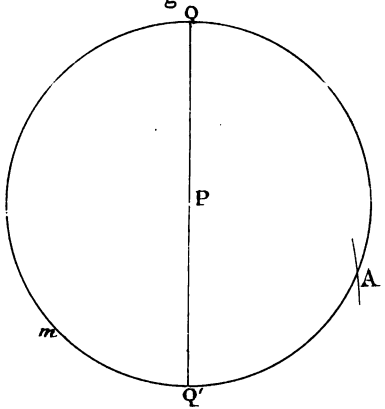
Fig. 13.



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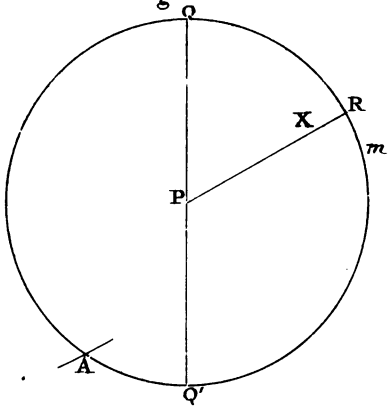

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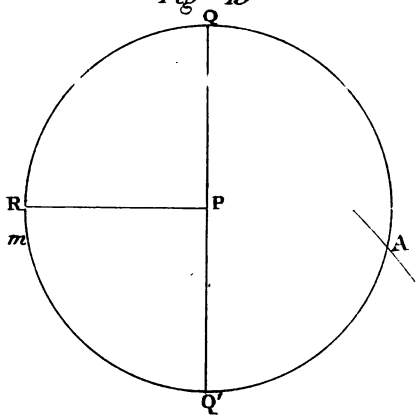


Fig 16

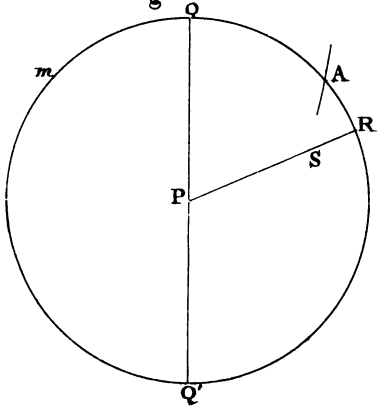
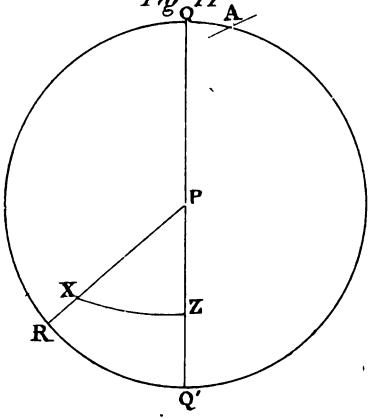


Fig 17



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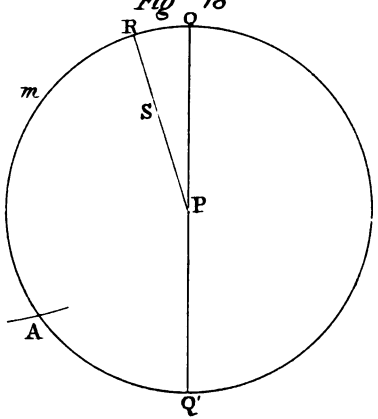

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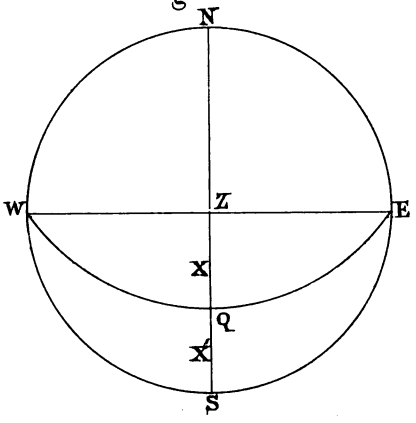
Fig 19



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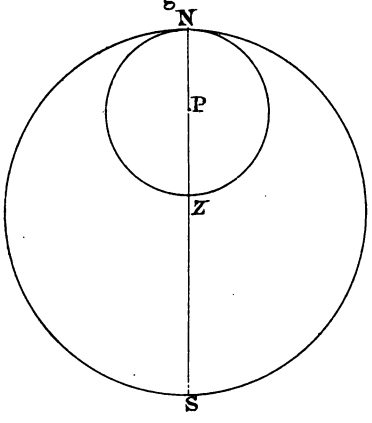

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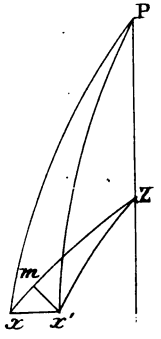

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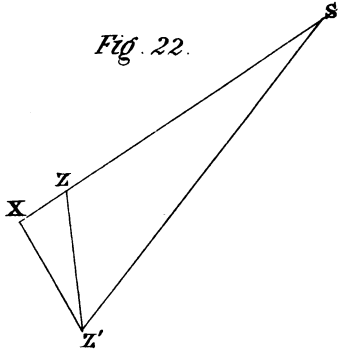

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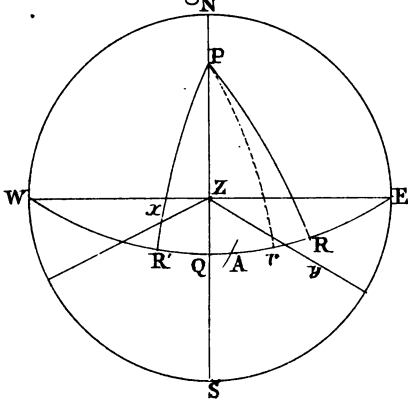

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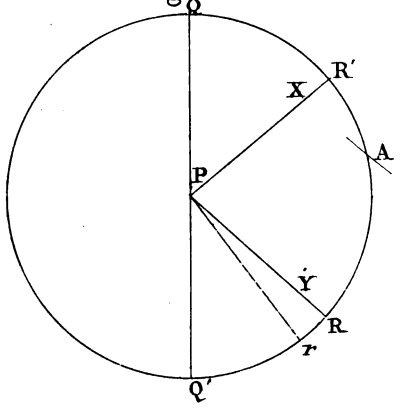

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Fig 25

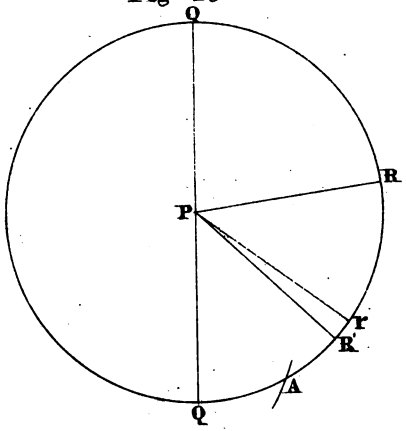
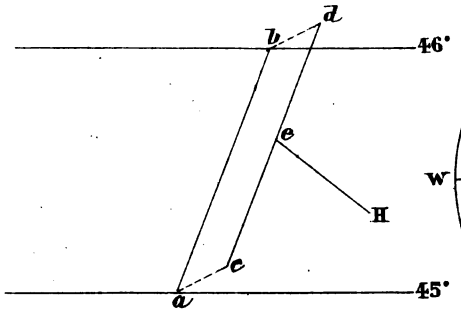


Fig 26



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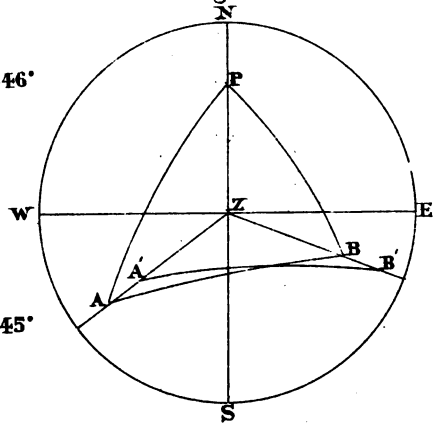

Fig 28

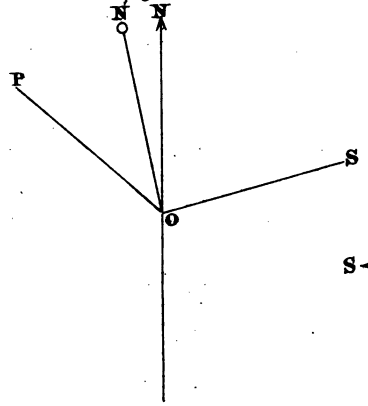

Fig 29

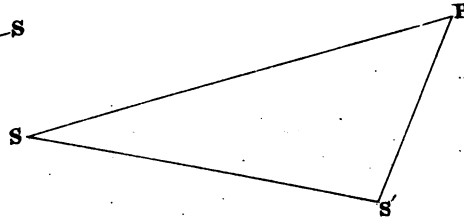

Fig 30

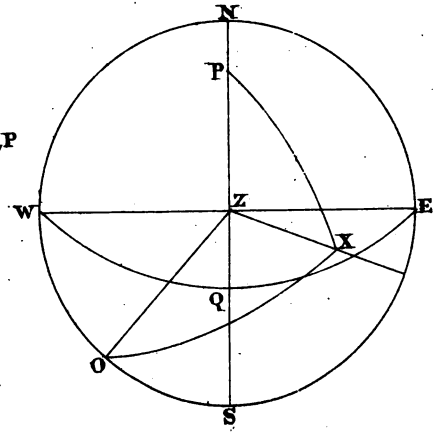

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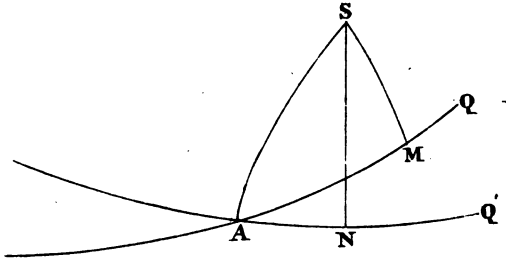
Fig 31



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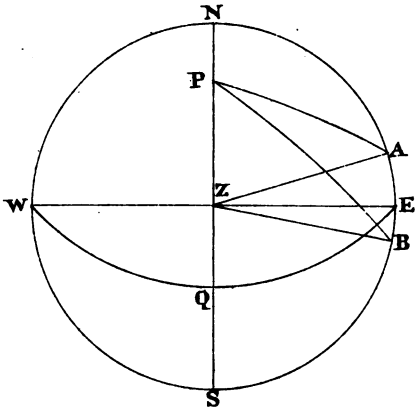


Fig 33

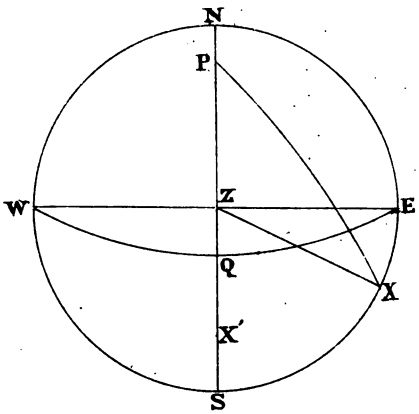
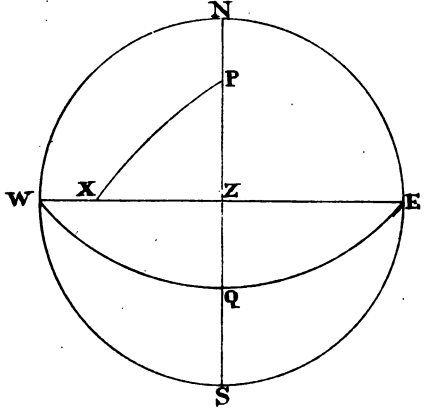


Fig 34



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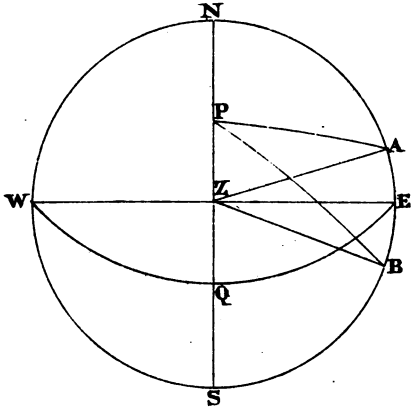

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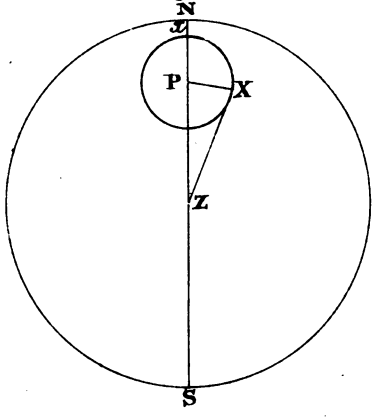

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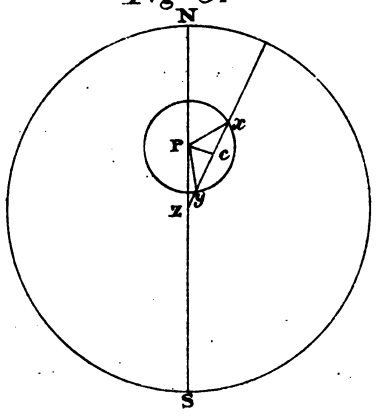
Fig 37



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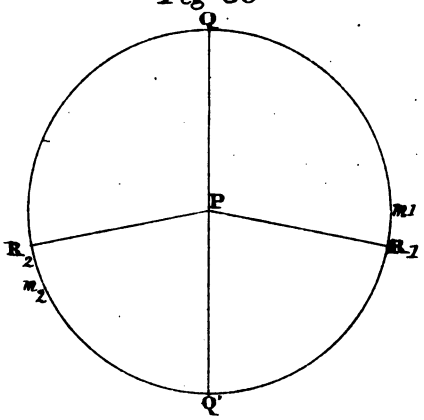

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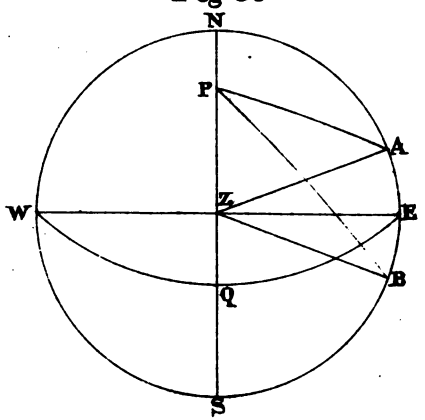

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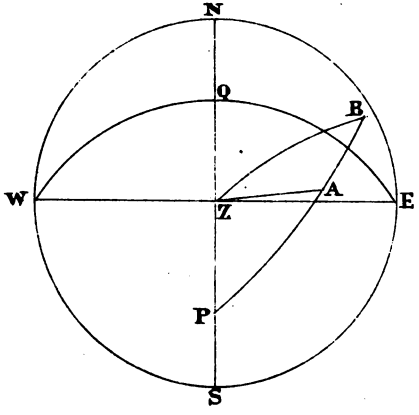

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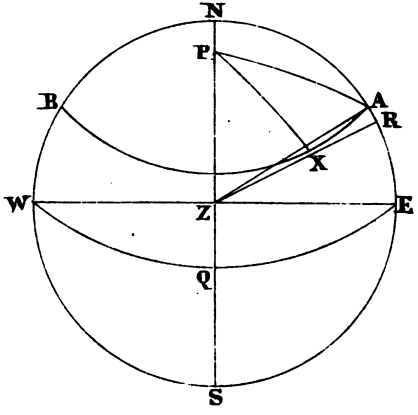

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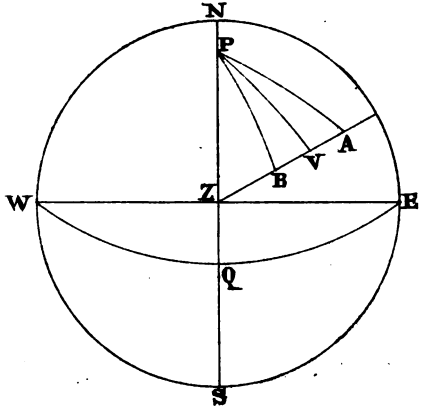

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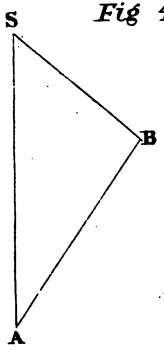

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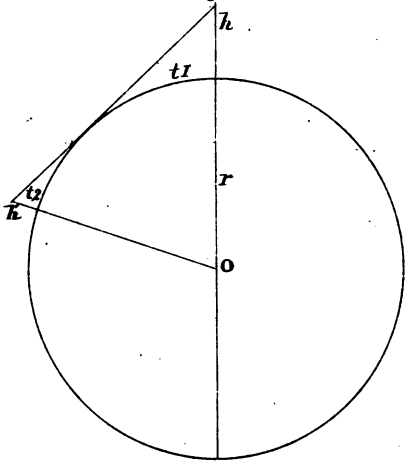

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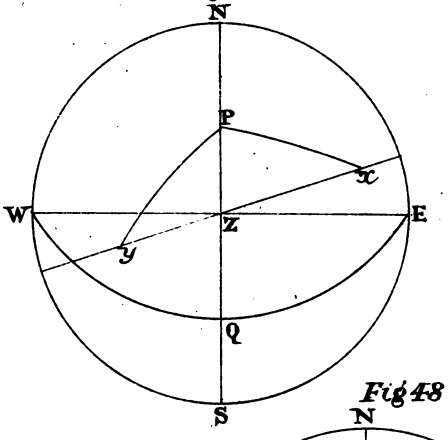

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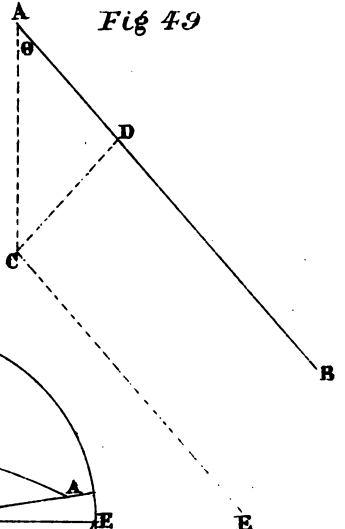

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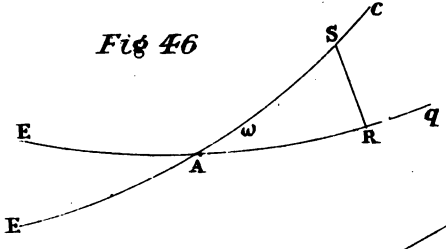

Fig 46

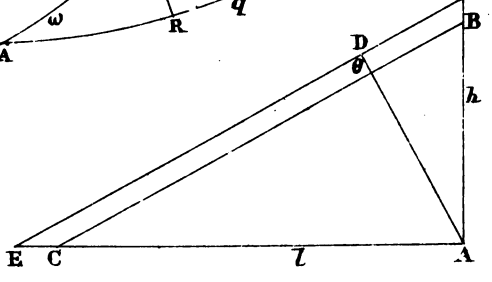

Fig 47

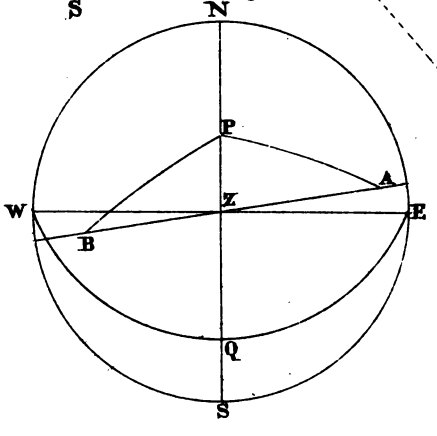

Fig 48

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Fig 50

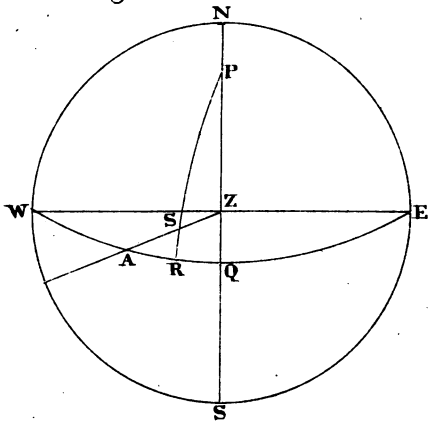
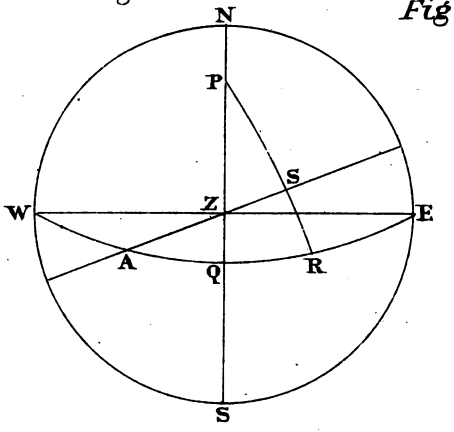


Fig 51



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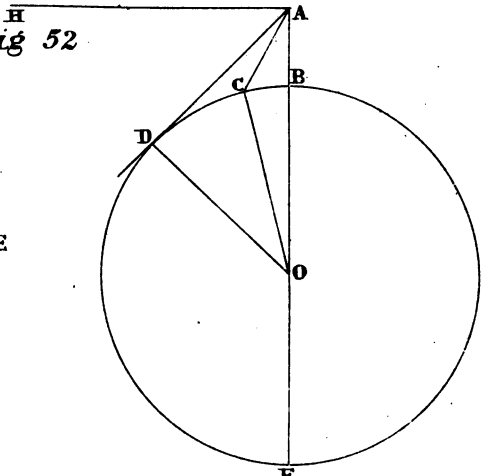

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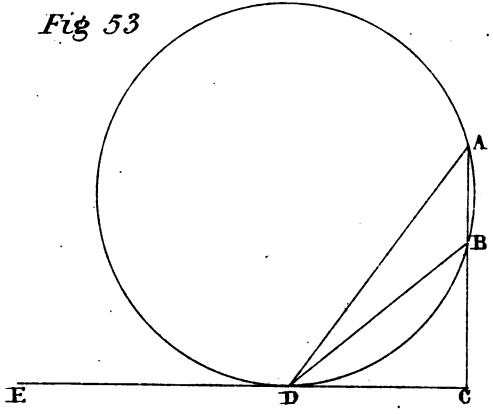
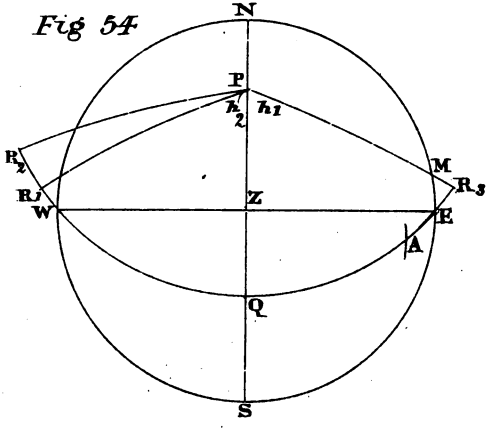


Fig 54



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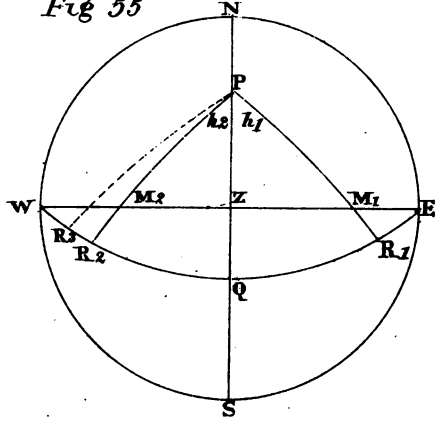

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PART II.

NAUTICAL ASTRONOMY.

I. TIME AND HOUR ANGLE.

1. Project a diagram on the plane of the Equator, and investigate the rule for finding the time of the meridian passage of a given star.

In longitude 53° W., a star was observed to be on the meridian at $7^h\ 56^m$ P.M.; find the R.A. of the star, having given from the Nautical Almanac, Sidereal Time at preceding G.M. Noon $13^h\ 8^m\ 26^s{\cdot}5$. (*November*, 1876.)

2. Find the *exact* Ship M.T. at which Aldebaran will bear West at a place in latitude 51° 30′ N. and longitude 107° 45′ E. on April 7th, having given from the Nautical Almanac, Dec. Aldebaran 16° 15′ 30″ N., R.A. Aldebaran $4^h\ 28^m\ 48^s$, R.A.M.S. at G.M. Noon, April 7th, $1^h\ 4^m\ 24^s$. (*April*, 1876.)

3. Explain the terms: First Point of Aries; Sidereal Time.

The Time of Transit of the First Point of Aries given in the Nautical Almanac for January 16th, is $4^h\ 18^m\ 11^s{\cdot}1$; find Sidereal Time at Greenwich Mean Noon on January 17th. (*October*, 1876.)

4. The Mean Time of Transit of the First Point of Aries given in the Nautical Almanac for August 1st is $15^h\ 18^m\ 35^s{\cdot}52$. Find the Equation of Time at Greenwich Mean Noon on August 1st, having given the Sun's Apparent Right Ascension $8^h\ 45^m\ 0^s{\cdot}12$. (*June*, 1879.)

5. The time of "Transit of the First Point of Aries" given in the Nautical Almanac for May 10th is $20^h\ 44^m\ 56^s{\cdot}2$. Find the mean time at which the star Altair (Dec. 8° 33′ N., R.A. $19^h\ 44^m\ 55^s{\cdot}$) will next pass the meridian of Greenwich above the horizon. (*May*, 1879.)

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6. Having given from the Nautical Almanac the mean time of Transit of the First Point of Aries $13^h\ 3^m\ 50^s{\cdot}21$; find exactly the mean time at a place in Longitude 81° E., when the star *Sirius* (R.A. $6^h\ 39^m\ 59^s$) will pass the meridian on the same Astronomical Day. (*September*, 1882.)

7. Find the exact ship Mean Time when the first point of Aries will pass the meridian of 85° E., having given from the Nautical Almanac the Right Ascension of the Mean Sun for the day $12^h\ 50^m\ 46^s{\cdot}3$. (*November*, 1882.)

8. The G.M.T. of the passage of a star over the meridian of 80° E. was $3^h\ 15^m\ 53^s$ on January 19th, the R.A. of the star being $4^h\ 28^m\ 49^s$. Find R.A. Mean Sun at G.M. Noon, January 20th. (*February*, 1877.)

9. The Sidereal Time at Mean Noon was $11^h\ 38^m\ 26^s{\cdot}83$, the Equation of Time $4^m\ 58^s{\cdot}01$ additive to Mean Time, and increasing $\cdot 88^s$ an hour, find the Sun's *Apparent* Right Ascension at Mean Midnight. (*March*, 1885.)

10. An observer in longitude 120° W. took the first observation for Star Equal Altitudes at $7^h\ 35^m\ 11^s$ by chronometer, about $9^h\ 10^m$ P.M. The R.A. of the Star was $3^h\ 19^m\ 47^s{\cdot}6$, the Sidereal Time at the preceding Greenwich Mean Noon $14^h\ 11^m\ 56^s{\cdot}5$, and the error of the chronometer $2^h\ 27^m\ 18^s$ fast on G.M.T. What will the chronometer show at the second observation? (*June*, 1885.)

11. Define Sidereal Time, and shew how, having given sidereal time at any instant, to find mean time at the same instant.

On August 8th, at a place in longitude 65° E., the time shewn by a sidereal clock was $15^h\ 17^m\ 6^s$, the error of the clock slow on sidereal time at the place being $1^h\ 46^m\ 10^s$. Required mean time at the instant, having given R.A.M.S. at G.M. Noon, August 8th, $9^h\ 6^m\ 20^s{\cdot}9$. (*February*, 1876.)

12. At a place in longitude 28° W., on March 25th, Altair was observed to rise at $5^h\ 56^m\ 7^s$ by chronometer. The error of the chronometer fast on G.M.T. was $3^h\ 8^m\ 7^s$; find the

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latitude of the place, having given from the Nautical Almanac Sidereal Time at G.M. Noon, March 24th, $0^h\ 7^m\ 18^s{\cdot}3$, R.A. Altair $19^h\ 44^m\ 50^s$, Dec. Altair 8° 32′ 50″ N. (*September*, 1878.)

13. At a place in longitude 27° 40′ E., about 3 P.M. on November 28th, a sidereal clock showed $19^h\ 31^m\ 56^s$ at $6^h\ 49^m\ 10^s$ by chronometer. The clock was known to be $5^m\ 27^s$ fast on sidereal time at the place. Find the error of the chronometer on G.M.T., having given from the Nautical Almanac, Sidereal Time at G.M. noon, November 28th, $16^h\ 29^m\ 0^s{\cdot}7$. (*April*, 1878.)

14. At a place in latitude 47° 30′ N. the First Point of Aries was on the meridian at $2^h\ 36^m\ 57^s$ A.M. Find the time shown by a sidereal clock, fast $8^m\ 5^s$ on local sidereal time, at sunrise on the same morning, having given the declination 14° 52′ 20″ N., and equation of time $4^m\ 47^s$ additive to apparent time. (*November*, 1878.)

15. The Sun was on the 6 o'clock hour circle, the equation of time $10^m\ 30^s$ subtractive from Apparent Time, and the Sidereal Time at Mean Noon was $12^h\ 42^m\ 49^s$; what was the Sidereal Time at the instant? (*November*, 1884.)

16. Find the time at which Saturn will rise on June 26th, at a place in latitude 45° 30′ S., longitude 108° E., having given from the Nautical Almanac,

R.A. Saturn, at G.M. Noon,	June 26th,	$22^h\ 41^m\ 18{\cdot}7^s$	
R.A. „ „	June 27th,	22	41 15·7
Sidereal Time „	June 26th,	6	19 48·6

Dec. Saturn, 10° 8′ S. (*September*, 1876.)

17. At a place in latitude 51° N., longitude 79° 30′ E., a Sidereal Clock showed $10^h\ 10^m\ 31^s$, the error of the clock on Sidereal Time at Greenwich being $2^h\ 1^m\ 17^s$ fast. What was the altitude at the instant of the star Regulus, the Right Ascension of which was $10^h\ 1^m\ 54^s$, and the declination 12° 33′ 40″ N.? (*March*, 1879.)

18. Explain the construction of the Table used in finding the Greenwich Date of the Moon's Meridian Passage.

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Find approximately the longitude of the place at which the Moon will be on the meridian at $4^h\ 59^m$ G.M.T. on May 19th, having given from the Nautical Almanac:

Mer. Pass. on May 18th	...	...	$7^h\ 53{\cdot}4^m$
„ „ 19th	...	...	8 37·8

(*May*, 1876.)

19. On July 17th, the Moon passed the meridian of 125° W. at $14^h\ 30{\cdot}7^m$ G.M.T. The Transit given in the Nautical Almanac for July 17th is $5^h\ 53{\cdot}6^m$; find approximately that for July 18th. (*June*, 1877.)

20. At what local time will the planet Saturn pass the meridian of a place in longitude 119° E. on July 4th, having given from the Nautical Almanac—

Mer. Pass., July 3rd	-	$16^h\ 38{\cdot}6^m$
„ July 4th	-	16 34·6

(*October*, 1877.)

21. The longitude of an observer was 15° E., the R.A. of a circumpolar star $16^h\ 2^m\ 27^s$, its hour angle $11^h\ 13^m\ 48^s$, and the R.A.M.S. at the preceding Greenwich Noon $19^h\ 40^m\ 36^s$; find, with the assistance of a diagram, the S.M.T. at the moment, and when the star will next pass the meridian. (*February*, 1885.)

22. In longitude 147° 30′ E. the time by chronometer of the first observation for equal altitudes was $10^h\ 15^m\ 23^s$, the equation of time $5^h\ 48^s$ subtractive from Mean Time, and the reputed error $58^m\ 37^s$ slow on G.M.T. Find approximately the chronometer time of the second observation. (*December*, 1884.)

II. THE SEXTANT.

1. Explain with a figure the theory of the Vernier of the Sextant.

The vernier of a sextant, the limb of which is divided to 10′, has 40 divisions, coinciding with 79 divisions on the limb, With what accuracy may the sextant be read off? (*May*, 1876.)

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2. The limb of a sextant is cut to 15′, and it is required to divide the Vernier so as to read to the nearest 20″. How many divisions should there be on the Vernier, and with how many divisions of the limb should its length coincide? (*September*, 1877.)

3. The arc of a sextant is divided to 15′; show how to form a vernier, so that the sextant may be read off to the nearest 10 seconds. (*December*, 1880.)

4. The limb of a sextant is divided to 15′, and 89 divisions on the limb are formed into 90 divisions on the vernier. Determine the accuracy to which the sextant can be read off.

5. If every inch on a barometer scale be divided into 20 parts, show how to form a vernier such that the reading of the instrument may be taken to the thousandth part of an inch. (*December*, 1883.)

III. MERIDIAN AND CIRCUM-MERIDIAN ALTITUDES.

1. In what latitude are the meridian altitudes of the Sun at the solstices complementary to each other? (*December*, 1878.)

2. What are circumpolar stars?

The declination of a star is 62° N.; between what limits of latitude is it circumpolar?

Find its meridian altitude above and below the pole in latitude 34° N. (*April*, 1880.)

3. Show how to determine accurately the mean time of the passage of a heavenly body over any meridian.

Under what circumstances is the meridian altitude not the maximum altitude of a heavenly body? (*April*, 1884.)

4. Explain the method of finding latitude by observation of a heavenly body near the meridian, above pole, without using estimated latitude.

In latitude by account 37° S., an observation of the sun (near the meridian) was taken at $0^h\ 58^m\ 16^s$ by chronometer. The Ship Mean Time, when sights were taken for longitude

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at $9^h\ 21^m\ 41^s$ by chronometer, was found to be $8^h\ 37^m\ 46^s$, and the ship had run in the interval 25 miles E. by S. Required the Sun's Hour Angle at the second time of observation, having given the Equation of Time (additive to Mean Time) $7^m\ 13^s$, and the rate of the chronometer 12 sec. losing. (*March*, 1877.)

5. *(a)* At a place in latitude 41° 50′ N. an error of $2^m\ 30^s$ was made in the hour angle. Find the resulting error in latitude, having given the azimuth of the body S. 12° E. (*September*, 1878.)

(b) If there be an error of $1^m\ 30^s$ in the Ship Time, compare the errors resulting when the bearings of the Sun are S. $1\frac{1}{4}$ pt. E. and S. $\frac{1}{4}$ pt. E. respectively. (*August*, 1876.)

6. Show how to find the latitude by the altitude of a heavenly body near the meridian and below the Pole, the approximate latitude being given.

(a) The R.A. of a Star being $5^h\ 27^m$, and that of the Mean Sun $6^h\ 58^m$, the longitude 75° E., and the chronometer $1^h\ 10^m$ fast on G.M.T., find the limits of time shown by chronometer when the star is within 20 minutes (mean time) of its inferior transit. (*September*, 1885.)

(b) Give approximately the limits of latitude in which this observation is possible in the case of the moon. (*June*, 1878.)

IV. LATITUDE BY DOUBLE ALTITUDE.

1. Under what circumstances in a Sun Double Altitude are your observations unaffected by the run of the ship in the interval? (*February*, 1884.)

2. Between the observations for Double Altitude a ship ran S.E. $\frac{1}{4}$ E. 18 miles and N. $\frac{3}{4}$ W. 7 miles, the bearing of the Sun at the first observation being E. $\frac{3}{4}$ S. Find the correction. (*February*, 1879.)

3. Between the observations for Double Altitude a ship ran E.S.E. 8 miles, and N.W. 3 miles, the bearings of the sun at the times of observation being N.E. and N.N.W. respectively.

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To which altitude should the correction be applied, and what is its amount, the latitude required being that of the first place of observation? (*June*, 1876.)

4. At 6^h A.M. and 11^h A.M. when the Sun bore N. 85° E. and S. 15° E. respectively, altitudes were observed for latitude, and during the interval the ship sailed N.E. ½ E. 14 miles, W. by N. 12 miles, and S. ¾ E. 10 miles. Find the correction for run, and show how to apply it, the latitude required being that at the first observation. (*August*, 1880.)

5. A ship steamed 9 knots S. by E. for 3½ hours between two forenoon observations, and the correction applied to the first altitude was—13′ 25″: what was the bearing of the Sun at the first observation? (*October*, 1884.)

6. The Sun's observed altitude was 37° 5′ shortly before noon. Corrected for Run it became 36° 57′ 30″, the ship having sailed in the interval 13 miles S. 50° E. What was the Sun's bearing at the time the observation was taken? (*October*, 1878.)

7. Required the exact polar angle in the following case of two stars, x and y, in latitude 33° N.:—

	Alt.	Dec.	R.A.	Bearing.
x,	64°	16° N.	$22^h\,29^m\,56^s$	S.W. b. W.
y,	27	8 S.	1 15 21	S.E. b. E.

The altitude of x was taken $1^h\,52^m\,21^s$ after that of y, as measured by a chronometer. (*April*, 1877.)

8. Altitudes of the same star were observed at $10^h\,46^m\,31^s$ and $2^h\,35^m\,59^s$, times by chronometer; find the polar angle in time exactly. (*March*, 1876.)

9. The R.A. of Altair is $19^h\,45^m\,6^s$ and of α Arietis $2^h\,0^m\,48^s$, the Polar Angle was $5^h\,58^m\,11^s$; what was the interval of mean time between the observations, and which star was taken first? (*September*, 1885.)

10. The R.A. of *Vega* is $18^h\,33^m\,0^s$, and that of *Markab* $22^h\,58^m\,59^s$; if the Polar Angle was $4^h\,50^m\,24^s$, what was the interval by chronometer between the observations, and which star was taken first? (*September*, 1884.)

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11. Altitudes of the planet Jupiter were observed on September 3rd, 1880, at $8^h\ 15^m\ 12^s$ P.M. and $11^h\ 3^m\ 50^s$ P.M. mean times.

Find the polar angle in time, the right ascension of Jupiter, as given in the Nautical Almanac for Greenwich Mean Noon on the 3rd $1^h\ 10^m\ 31^s{\cdot}4$, and on the 4th, $1^h\ 10^m\ 13^s{\cdot}1$. (*May*, 1880.)

12.*(a)* In a Double Altitude of the planet Mars the elapsed time, as measured by a chronometer keeping mean time, was $4^h\ 0^m\ 30^s$, and the increase of the planet's Right Ascension in the interval was 27·2 seconds. Required the Polar Angle. (*August*, 1875.)

(b) In a Double Altitude of a planet, the elapsed time, as measured by a chronometer, was $3^h\ 56^m\ 30^s$, and the decrease of the planet's Right Ascension was 31·5 seconds. Required the Polar Angle. (*June*, 1878.)

V. SUMNER'S METHOD OF FINDING LATITUDE.

1. Explain Sumner's method of finding a ship's position by Double Altitude ; and show that the bearing of the body observed is at right angles to the line of position. (*March*, 1884.)

2. If one observation only can be obtained, is the resulting line of position ever of any practical use ? (*March*, 1878.)

3. An altitude of the Sun was taken one forenoon, and worked out with assumed latitudes of 35° 30′ N. and 35° 50′ N. respectively. The first gave a longitude of 14° 24′ W. and the second of 14° 5′ W. Find the direction of the line of position. (*October*, 1884.)

4.*(a)* An observation taken about 10^h A.M. gave with latitude 51° N., longitude 15° 46′ W., with latitude 52° N., longitude 11° 51′ W. Required the Sun's bearing at the time. (*May*, 1879.)

(b.) At the first observation one forenoon, an assumed latitude of 50° 30′ N. gave a longitude of 15° 30′ W., but a latitude of 50° 40′ N. gave a longitude of 15° 23′ W.; what was the bearing of the Sun ? (*September*, 1884.)

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5. Working out an altitude with a latitude of 44° 50′ N., a longitude of 27° 30′ W. was obtained, and the sun's bearing was S. 60° E. What longitude would be obtained by using a latitude of 45° 10′? (*March*, 1885.)

6. A ship, the latitude of which was uncertain, but was known to lie between 45° N. and 46° N., observed the altitude of the sun for chronometer, when its bearing was E.S.E. (true). After the ship had sailed E.N.E. (true) 12 miles, a known headland appeared bearing S.E. b. S. by compass (var. 2 pts. W., dev. ½ pt. E.)

Shew, with a diagram, how Sumner's Method may be applied to find the position of the ship. (*Beaufort, April*, 1875.)

VI. LONGITUDE BY CHRONOMETER.

1. Investigate a formula for finding the error in hour angle resulting from a given small error in the altitude of a heavenly body.

In lat. 35° S. an error of 0′ 50″ in altitude produced an error of 7 seconds in hour angle. What was the bearing of the Sun at the time? (*September*, 1877.)

2. At a certain place when the sun was on the prime vertical an error of 2′ in altitude produced an error of 10·4 seconds in hour angle. What was the sun's azimuth, when the same error of altitude produced an error of 16·15 seconds? (*October 29th*, 1877.)

3. What is the best time for observing an altitude for longitude, and why?

Compare the errors in the apparent time caused by an error in an observed altitude of the sun when his bearings are E. by S. and S.E.b.S. respectively. (*February*, 1880.)

4. If e and e_1 be the errors in hour angle corresponding to an error of a minutes in the altitude of the Sun observed when the bearings were S. 60° E. and S. 30° E. respectively, show that—

$$e_1 = e\sqrt{3}.$$ (*April*, 1883.)

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5. Show that at a place in latitude 60°, when the Sun is on the prime vertical, the error in hour angle (expressed in time) is to the error in altitude as 2 : 15. (*April*, 1882.)

6. Show that in any given latitude the errors in hour angle corresponding to a given error in altitude, when the bearings of the body are S. 15° E. and S. 75° E. respectively, are as $2 + \sqrt{3}$ to unity. (*June*, 1881.)

VII. LONGITUDE BY LUNAR.

1. Why, in a Lunar observation, is the apparent altitude of the Sun greater, and that of the Moon less, than the true altitude? (*February*, 1884.)

2. Explain the principle upon which the Greenwich Mean Time is obtained in a Lunar observation, and show how you would determine, by means of the Nautical Almanac, the body best suited for observation. (*December*, 1877.)

3. Show that the observations required for a Lunar afford the means of obtaining the latitude as well as the longitude, without using the Right Ascensions of the bodies observed. (*November*, 1878.)

4. Upon what principle are stars selected in the Nautical Almanac for lunar distances? (*April*, 1884).

5. Define a Proportional Logarithm, and prove the rule for finding G.M.T. in a lunar observation by means of these logarithms.

Find by proportional logarithms the True Distance between the Sun and Moon at $9^h\ 46^m\ 50^s$ G.M.T., having given from the Nautical Almanac

Distance at	$9^h\ 0^m$	..	85° 47′ 50″
„	12 0	..	87 19 24

(*June*, 1876.)

6. In lat. 15° 10′ S. the true altitude of the Sun's centre was 7° 51′ 30″ (W. of Meridian). Required the true altitude of the Moon's centre, having given

App. R.A. Sun $12^h\ 38^m\ 9^s$. App. Dec. Sun 4° 7′ 15″ S.
R.A. Moon 15 51 59. Dec. Moon 24 35 15 S.

(*September*, 1877.)

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VIII. ON VARIATION AND DEVIATION OF COMPASS.

1. The bearing of a mountain peak by the azimuth compass was N. 50° W., and the ship's head by the same compass N. 77° E.; by the standard compass the ship's head was N. 74° E. What was the bearing of the peak by the standard compass? (*February*, 1877.)

2. A ship wished to reach a port 120 miles E.N.E. of her. After steaming 100 miles she found that, the variation of compass having been applied the wrong way, she was still 50 miles from her port, and to the southward of it. Required the variation. (*June*, 1877.)

3. Prove the rule for finding the variation of the compass by observing the Amplitude of a heavenly body.

Why is the result of this observation less reliable in high latitudes than in low? (*October*, 1880.)

4. The true altitude of the Sun's centre one forenoon was 38°, declination 10° N., angle to a horizontal object on the right 96°, Compass bearing of the object S. 20° W., latitude of observer 35° N. Find the error of the compass. (*September*, 1885.)

5. In latitude 43° 56′ S., at $3^h\ 35^m$ P.M. Ship Apparent Time, an object *O* in the horizon was observed to be 94° 36′ distant from the Sun's nearest limb: *O* being to the right of the Sun and E. of Meridian. Find the true bearing of *O*, having given:

Sun's Semidiameter 16′ 18″

Sun's Declination 21° 42′ S. (*October*, 1876.)

6. In latitude 31° 37′ N., at $3^h\ 41^m$ P.M. Ship Apparent Time, an object *O* in the horizon was observed to be 85° 36′ distant from the Sun's nearest limb, *O* being to the right of the Sun and West of Meridian. Find the true bearing of *O*, having given

Sun's Semidiameter 15′ 53″

Sun's Declination 8° 14′ 45″ N.

(*October*, 1878.)

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7. Shew how Deviation may be determined at sea by observation of the Sun's Amplitude. Explain, with a diagram, the effect of Refraction upon this observation. (*June*, 1875.)

8. In finding Variation of Compass, is it advisable to observe the Amplitude of the Moon for that purpose?

9. In latitude 59° 10′ N., the Amplitude is double the declination. Find the Variation, having given the Compass bearing of the Sun E. 17° N., and the deviation 5° 30′ W. (*February*, 1879.)

IX. MISCELLANEOUS PROBLEMS.

1. Find the time on November 7th (Civil Day) when at a place in lat. 56° 21′ S., long. 108° W., the star Regulus will have an altitude equal to its declination, before reaching the meridian; having given from the Nautical Almanac—

R.A. Regulus $10^h\ 1^m\ 55^s.7$; Dec. Regulus 12° 33′ 30″ N.; R.A.M.S. at G.M. noon Nov. 6th, $15^h\ 2^m\ 16^s.4$. (*May*, 1878.)

2. Show that if the declination of a star be equal to its longitude, its latitude must be equal to its Right Ascension. (*June*, 1878.)

3. At a certain place in North latitude Rigel (Dec. 8° 20′ S., R.A. $5^h\ 8^m$) and Pollux (Dec. 28° 19′ N., R.A. $7^h\ 38^m$) rise at the same instant. Required the latitude. (*April*, 1876.)

4. At a certain place the Sun rose at $6^h\ 15^m$ A.M., and the meridian zenith distance was three times as great as the latitude. Find the latitude. (*November*, 1875.)

5. Investigate a formula for finding the length of day at a given place on a given day.

What deductions may be drawn from this formula with regard to the length of day under the following circumstances?—(1.) When the latitude=0. (2.) When the declination is the complement of the latitude, and of the same name. (3.) When the declination is greater than the complement of the latitude. (*December*, 1877.)

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6. Write down the formula for finding the time when, at a given place, a heavenly body of known declination will reach the prime vertical.

Interpret the results which this formula gives (1) when the declination is zero, (2) when the declination and latitude are of the same name, and the declination is the greater, and (3) when the declination and latitude are of the same name and equal to each other. (*April*, 1879.)

7. At a certain place two stars rise together. The declination of one is 15° N., and of the other 15° S., and the first reaches the meridian $1^h\ 59^m\ 40^s$ (mean time) before the other. Find the latitude of the place. (*May*, 1884.)

8. If at a certain place in north latitude the Sun rises on the longest day at 4.40 A.M., at what time will it rise on the shortest day, and what will be its amplitude? (*November*, 1884.)

9. The inferior meridian altitude of a circumpolar star is equal to its maximum azimuth, and its declination is 75° N. Find the latitude of the place. (*May*, 1882.)

10. The inferior meridian altitude and maximum azimuth of a star north of the equator are each 30°; find its declination, and the latitude of the observer. (*September*, 1884.)

11. The altitude of a circumpolar star, the declination of which was known to be 78° 30′, was observed twice on the same evening, the bearing on each occasion being N.N.E., while the difference of altitude was 6° 30′. Find the latitude of the place. (*November*, 1876.)

12. The maximum azimuth of a circumpolar star is twice the inferior meridian altitude, and the latitude of the place is 45° N. Find the declination of the star. (*December*, 1882.)

13. If the times of Sunrise and Sunset, given in the Almanac for November 9th, be $7^h\ 2^m$ A.M. and $4^h\ 26^m$ P.M. respectively, find approximately the equation of time. (*April*, 1878.)

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14. The declination of a star is 21° N., and its Right Ascension $17^h\ 5^m\ 36^s$. At a place North of the equator it rises with the Sun when the Right Ascension of the latter is $14^h\ 50^m\ 27^s$, and its declination 15° 10′ S. Find the latitude of the place. (*March*, 1879.)

15. At a place in South latitude the altitude of Fomalhaut (Dec. 30° 16′ S.) was observed, and found to be 42° 21′ 30″. After an interval of $7^m\ 47^s$, corresponding to the difference of Right Ascension between the stars, the altitude of α Pegasi (Dec. 14° 33′ N.) was taken, and found to be 10° 0′ 30″. Required the latitude. (*April*, 1878.)

16. At a place in North latitude the altitude of Castor (Dec. 32° 9′ 30″ N.) was observed to be 30° 37′, and after an interval of sidereal time corresponding to the difference of Right Ascension of the two bodies, that of Procyon (Dec. 5° 32′ 15″ N.) was also taken, and found to be 30° 1′. Required the latitude. (*October*, 1877.)

17. Under what circumstances does the Sun's shadow change its direction of rotation in the course of the day?

The Sun rose at $5^h\ 52^m\ 42^s$, and its shadow moved backwards for $1^h\ 2^m\ 56^s$. Find the latitude. (*February*, 1884.)

18. In latitude 18° N., find the time at which Pollux (Dec. 28° 18′ N.) will appear stationary in azimuth, after crossing the meridian at 5^h P.M., S.M.T. (*March*, 1885.)

19. In latitude 16° 40′ N., the Sun bore N. 68° 30′ E. twice in the same forenoon, the declination being 23° 18′ N. Find the altitudes. (*May*, 1880.)

20. Investigate an expression for finding the dip of the sea horizon for a given altitude of the eye.

How far can a ship's nettings (20 feet above the water-line) be seen by an observer, whose eye is 30 feet above the sea level? (*February*, 1885.)

21. From a vessel at anchor, a ship, whose nettings were just visible, bore S.; 50 minutes later her water-line was

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visible, and her bearing S.E. Find the course and speed of the ship during the interval.

Height of the nettings above the water-line 18 feet, of the observer's eye 26 feet. (*April*, 1884.)

22. If h be the height of the observer's eye above the water-line, and h' the height of an object, show that the distance at which the object is visible is nearly $\sqrt{2r}.\ \{\sqrt{h} + \sqrt{h'}\}$.

If the top of a funnel of a steamer 36 feet above its water-line is just visible at a distance of 11·7 nautical miles by an observer whose eye is 25 feet above the water-line, what is the diameter of the earth? (*June*, 1884.)

23. At a place in latitude 55° N. the altitude of a star (Declination 18° N.) was observed when E. of meridian to be 9°. What was its altitude when upon the same vertical circle W. of meridian? (*March*, 1879.)

24. Having given the obliquity of the ecliptic $\tan^{-1}\frac{3}{7}$, find the declination of the Sun when the declination and Right Ascension are complementary to each other. (*June*, 1879.)

25. At a place in North latitude the shadow of a vertical object at noon was equal to its length, the declination of the Sun being 10° 30′ S. At what time in the afternoon will the length of the shadow be to the height of the object as 3 to 1? (*February*, 1876.)

26. The Sun's altitude being 29° 47′, the length of a shadow of a perpendicular stick was observed to be 3·56 feet. Find the length of the longest shadow which could be obtained by inclining the stick. (*February*, 1878.)

27. At a certain place in North latitude, two sticks, $4\frac{1}{4}$ feet and $6\frac{1}{2}$ feet high respectively, were placed vertically in the ground, at a distance of 18 feet. In the forenoon the shadow of the first just reached the foot of the second; in the afternoon the extremity of the shadow of the second fell at the foot of the first. Find the latitude of the place, having given the declination of the Sun 15° N. (*December*, 1878.)

28. The shadows of two walls, which are at right angles

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to one another, and are 13 feet and 11 feet in height respectively, were observed at noon to be 15 feet and 13·5 feet broad. Required the altitude of the Sun and the inclination of the walls to the meridian. (*May*, 1876.)

29. The Sun rose at $4^h\ 49^m$ A.M., and the latitude was double the declination; find both. (*June*, 1886.)

30. Having given the altitude of the Sun at 4^h P.M. apparent time 31° 45′, and the hour angle at setting $6^h\ 45^m$, find the declination and latitude. (*December*, 1880.)

31. At a place in latitude $l°$ N. the Sun rose S. 60° E., the declination being 15°. Show that $\cos. l = \frac{\sqrt{3}-1}{\sqrt{2}}$. (*Sept.*, 1881.)

32. What is approximately the maximum declination of the Sun? Show, with the assistance of a diagram, that the declination changes more rapidly at the equinoxes than at the solstices. (*December*, 1884.)

33. At a certain place a star, having North declination d, when due West had an hour angle $\tan^{-1}\frac{3}{4}$. At the same place another star, of South declination $2d$, was setting, and its hour angle was $\cot.^{-1}\frac{3}{4}$. Required the latitude of the place and declination of the stars. (*April*, 1879.)

34. The latitude of a place is 15° N.; find the declination of the Sun, when, on a day in April, the ecliptic is perpendicular to the horizon at $1^h\ 40^m\ 30^s$ P.M. apparent time. (*August*, 1882).

35. In the month of May, at a place in latitude $l°$ N., longitude 48° W., the ecliptic was perpendicular to the horizon when a chronometer showed $1^h\ 2^m\ 48^s$. The declination of the Sun was 17° N. and the Equation of Time $3^m\ 40^s$ additive to Mean Time. Find l, the chronometer being slow on Greenwich Mean Time $1^h\ 3^m\ 32^s$. (*February*, 1882.)

36. The angle between a target laid out from a ship and the horizon, as taken by an observer 96 feet above the waterline, is 1° 37′ 20″. Find approximately the distance of the target from the ship. (*February*, 1884.)

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37. A lighthouse 110 feet high stands on the top of a cliff 250 feet high. At what distance from the foot of the cliff will the lighthouse subtend the greatest possible angle, and what is the value of that angle? (*Beaufort, November*, 1878.)

38. At a certain place in North latitude the Moon rose $36^m\ 0^s$ after sunset. Required the latitude, having given from the Nautical Almanac:

Sun's App. R.A. (at setting)		$11^h\ 36^m\ 20^s$ (Var. in 1 hour 9^s).
Sun's App. Dec.	„	$2°\ 25'$ N.
Moon's R.A. (at rising)		$0^h\ 37^m\ 10^s$.
Moon's Dec.	„	$4°\ 30'$ N. (*Beaufort, Oct.*, 1875).

39. In a certain latitude North the star α Hydræ (Dec. $8°\ 7'$ S., R.A. $9^h\ 21^m$) was observed to rise $2^h\ 54^m\ 5^s$ later than Procyon (Dec. $5°\ 32'$ N., R.A. $7^h\ 33^m$), as measured by a chronometer; find the latitude. (*Beaufort, April* 1875.)

40. The interval by chronometer between the passages of the Moon over the Prime Vertical was $9^h\ 10^m$, the declination at meridian passage was 17° N., hourly change $6'\ 30''$ increasing, and the hourly change in R.A. was $2^m\ 24^s$. Find the latitude. (*Beaufort, March*, 1886.)

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PART II.

NAUTICAL ASTRONOMY.

ANSWERS AND HINTS TO SOLUTION.

I.—TIME AND HOUR ANGLE.

1. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 29.

See fig. 1.

Let QQ′ be the equator, QPQ′ the meridian, P being the Pole. Let X represent the star upon the meridian.

Measure Q′m westward, equal to $7^{h.}\ 56^{m.}$ Then Q′m represents the mean time.

From m through Q measure $13^{h.}\ 8^{m.}\ 26{\cdot}5^{s.}$, which is approximately the R.A.M. Sun, and we can fix the position of A, the first point of Aries—

From the figure

$$AQmQ' = AQm + mQ'$$

Or—

R.A. Star = R.A.M.S. + Ship M.T.

G	D.		H.	M.	S.
$7^{h.}$	$56^{m.}$	R.A.M.S. at G.M. Noon .	13	8	26·5
3	32			1	48·4
					4·6
11	28	R.A.M.S. (corrected) . .	13	10	19·5
		Ship M.T.	7	56	0
		R.A. Star	21	6	19·5

Ans., $21^{h.}\ 6^{m.}\ 19{\cdot}5^{s.}$

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2. *See fig.* 2.

Let QQ′ be the equator; QPQ′ the meridian; X the place of the body; R the point where the circle of declination through X cuts the equator.

By means of the Right Ascension of the Star we can fix the position of the first point of Aries, and thence that of the Mean Sun.

Let ZX be the vertical circle through X, so that PZ is the co-latitude.

First, to find the hour angle of the star.

In the triangle PZX,

$$\text{Cos. P} = \text{tan. PZ. cot. PX.}$$
$$= \text{cot. } l. \text{ tan } d.$$

Whence P or Q′R = $5^{h.}\ 6^{m.}\ 17^{s.}$

Then from the figure—

$$Q'm = Q'R + AR - Am.$$

Or—

Ship M.T. = H.A. Star + R.A. Star − R.A.M. Sun.

Using the approximate value of R.A.M. Sun given, viz.— $1^{h.}\ 4^{m.}\ 24^{s.}$, we obtain $8^{h.}\ 30^{m.}\ 45^{s.}$ Ship M.T. nearly.

Correcting for the increase of R.A.M. Sun in $1^{h.}\ 20^{m.}$, the Greenwich Date, we have for the true Ship Mean Time $8^{h.}\ 30^{m.}\ 32^{s.}$

Ans. $8^{h.}\ 30^{m.}\ 32^{s.}$ P.M.

3. *See fig.* 3.

Let QQ′ be the equator; P the pole; QPQ′ the celestial meridian.

Let A, the first point of Aries, be upon the meridian, coinciding with Q′.

Since we know the Ship M.T., we can fix the position of m.

Then AQm=R.A.M. Sun=$24^{h.} - Am = 24^{h.} - 4^{h.}\ 18^{m.}\ 11{\cdot}1^{s.}$
$= 19^{h.}\ 41^{m.}\ 48{\cdot}9^{s.}$

Thus Sidereal Time at next Mean Noon, or, which is the same thing, R.A.M. Sun at next Mean Noon = $19^{h.}\ 41^{m.}\ 48{\cdot}9^{s.}$ + the correction for the interval which has to elapse=$19^{h.}\ 41^{m.}\ 48{\cdot}9^{s.}$ + $3^{m.}\ 14{\cdot}1^{s.} = 19^{h.}\ 45^{m.}\ 3^{s.}$

Ans., $19^{h.}\ 45^{m.}\ 3^{s.}$

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4. *See fig.* 4.

Let PR be the circle of declination passing through the real Sun S.

Having given the time of Transit of the First Point of Aries, we know that at $15^h.\ 18^m.\ 35{\cdot}52^s.$ on August 1st, R.A.M. Sun was $8^h.\ 41^m.\ 24{\cdot}48^s.$

Therefore at Noon on August 1st it was $8^h.\ 41^m.\ 24{\cdot}48^s.$, less the augmentation for $15^h.\ 18^m.\ 35{\cdot}52^s.$, or $8^h.\ 41^m.\ 24{\cdot}48^s. - 2^m.\ 30{\cdot}9^s.$, that is $8^h.\ 38^m.\ 53{\cdot}58^s.$

Then, from the figure—

$Q'R = AR - AQ'$ or $AR - Am$, since m is on the meridian,

= R.A. Apparent Sun − R.A.M. Sun.

$= 8^h.\ 45^m.\ 0{\cdot}12^s. - 8^h.\ 38^m.\ 53{\cdot}58^s.$

Ans., $6^m.\ 6{\cdot}54^s.$ additive to apparent time.

5. *See fig.* 5.

At $20^h.\ 44^m.\ 56{\cdot}2^s.$ on May 10th, the R.A.M. Sun was $3^h.\ 15^m.\ 3{\cdot}8^s.$ Having then the star upon the meridian, and knowing the R.A.M. Sun, we can fix the positions of the first point of Aries, and of the Mean Sun.

From the figure—

$Q'Qm = Q'QA - Am.$

= R.A. Star − R.A.M. Sun.

$= 19^h.\ 44^m.\ 55^s. - 3^h.\ 15^m.\ 3{\cdot}8^s.$

$= 16^h.\ 29^m.\ 51{\cdot}2^s.$ nearly.

Thus the approximate time of the star's transit is $16^h.\ 29^m.\ 51{\cdot}2^s.$ on May 11th.

To find the *exact* time, we must correct the value of R.A.M. Sun for the interval between $20^h.\ 44^m.\ 56^s.$ on May 10th, and $16^h.\ 29^m.\ 51^s.$ on May 11th, and repeat the operation with the corrected Right Ascension.

The time required will be found to be $16^h.\ 26^m.\ 37^s.$ nearly.

Ans., at $16^h.\ 26^m.\ 37^s.$ on May 11th.

6. *See fig.* 6.

At $13^h.\ 3^m.\ 50{\cdot}21^s.$ G.M.T. the R.A.M. Sun was $10^h.\ 56^m.\ 9{\cdot}79^s.$ Hence we can fix the position of A and m.

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From the figure—

$$Q'Am = Q'A + AQm.$$
$$= \text{R.A. Star} + 24^h - \text{R.A.M. Sun},$$
$$= 19^h\ 43^m\ 49^s \text{ nearly.}$$

Since the longitude is 81° E., the Greenwich date will be found to be $14^h\ 20^m$.

The R.A.M. Sun made use of was that for $13^h\ 3^m\ 50^s$.

We must therefore correct R.A.M. Sun for $1^h\ 16^m$, and repeat the operation, when it will be found that the ship M.T. is $19^h\ 43^m\ 37^s$ nearly.

Ans., $19^h\ 43^m\ 37^s$.

7. *See fig.* 7.

From the figure—

$$Am = 24^h - AQm = 24^h - \text{R.A.M. Sun},$$
$$= 11^h\ 9^m\ 13{\cdot}7^s \text{ nearly.}$$

We can now obtain the Greenwich date, which is $5^h\ 29^m$ nearly, and repeating the process with the corrected value of the R.A.M. Sun, we shall obtain the correct Mean Time of Transit $11^h\ 8^m\ 19{\cdot}7^s$.

Ans., $11^h\ 8^m\ 19{\cdot}7^s$.

8. *See fig.* 8.

The Ship M.T. of Star's transit is $3^h\ 15^m\ 53^s + 5^h\ 20^m = 8^h\ 35^m\ 53^s$.

We can therefore fix the position of m, the Mean Sun.

From the figure—

$AQ'Qm = AQ' + Q'Qm = \text{R.A. Star} + 24^h - \text{Ship M.T.} = 19^h\ 52^m\ 56^s$.

This is the R.A.M. Sun at G.M.T. $3^h\ 15^m\ 53^s$ P.M. To find its value at Noon the next day, we must correct for $20^h\ 44^m\ 7^s$, when the corrected quantity will be $19^h\ 56^m\ 20{\cdot}4^s$.

Ans., $19^h\ 56^m\ 20{\cdot}4^s$.

9. *See fig.* 9.

Let PR be the circle of declination, passing through S, the real sun.

From the figure,

$$AR = Am - Rm.$$

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Or

Sun's apparent right ascension = R.A.M. Sun − Equation of time.

	H. M. S.		M. S.
R.A.M. Sun at noon .	11 38 26·83	Equation of time at noon	4 58·01
Correction for 12 hours.	1 58·27	Correction for 12 hours .	10·56
R.A.M. Sun at midnight	11 40 25·10		5 8·57

Thus Apparent Right Ascension = $11^{h.}\ 35^{m.}\ 16{\cdot}5^{s.}$ nearly.

Ans., $11^{h.}\ 35^{m.}\ 16{\cdot}5^{s.}$

10. *See fig.* 10.

First to find the star's hour angle when observed east of meridian.

G.D.		H. M. S.	R.A.M.S.
H. M.	Time by chronometer	7 35 11	
9 10	Error fast - -	2 27 18	H. M. S.
8 0		5 7 53	14 11 56·5
		12 0 0	2 47·56
17 10			1·15
	G.M.T. - - -	17 7 53	·14
	Longitude - -	8 0 0	14 14 45·35
	Ship M.T. - -	9 7 53	

From the figure,

$Q'QR = Q'm + Am - AR.$

= Ship M.T. + R.A.M.S. − R.A. Star.

	H. M. S.
Ship M.T. - -	9 7 53
R.A.M.S. - -	14 14 45·4
R.A. Meridian -	23 22 38·4
R.A. Star - -	3 19 47·6
Hour Angle of Star	20 2 50·8

Thus the star was distant $3^{h.}\ 57^{m.}\ 9{\cdot}2^{s.}$ from the meridian. After an interval of sidereal time equal to twice this hour angle, it will have the same altitude west of meridian. If, therefore, we express that interval in mean solar time, and add it to the time shewn by chronometer at the first observation, we shall obtain the chronometer time at the second observation.

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	H.	M.	S.
Time by chronometer - -	7	35	11
Interval in mean solar time -	7	53	1
Time by chronometer required -	15	28	12

Ans., at $3^{h.}\ 28^{m.}\ 12^{s.}$

11. *See Jeans' Nautical Astronomy and Navigation*, Part II., page 27. *See fig.* 11.

	H.	M.	S.
Time by clock -	15	17	6
Error slow - -	1	46	10
Sidereal time - -	17	3	16

From the figure,

$$Q'm = AQ' - Am.$$

Or, Mean time = Sidereal time − R.A. Mean Sun.

$= 17^{h.}\ 3^{m.}\ 16^{s.} - 9^{h.}\ 6^{m.}\ 21^{s.} = 7^{h.}\ 56^{m.}\ 55^{s.}$ nearly.

Having now Ship M.T. approximately, and the longitude 65° E., we may obtain the correction for R.A.M. Sun; working with the value so corrected, we may determine exact Ship M.T., which will be found to be $7^{h.}\ 56^{m.}\ 19{\cdot}5^{s.}$

Ans., $7^{h.}\ 56^{m.}\ 19{\cdot}5^{s.}$.

12. *See fig.* 12.

If the hour angle of Altair be known, we may find l, the latitude from the formula—

$$\text{Cos. } h = \pm \tan l \tan d.$$

To find the hour angle,

	H.	M.	S.	R.A.M.S. H.	M.	S.
Time by chronometer -	5	56	7			
	12	0	0			
	17	56	7			
Error fast - - -	3	8	7			
				0	7	18·3
G.M.T. - - -	14	48	0		2	17·9
Longitude - - -	1	52	0			7·9
Ship M.T. - - -	12	56	0	0	9	44·1

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From the figure,

$$Q'QR = Q'Qm + Am + AR.$$

Or, Star's H.A. = Ship M.T. + R.A.M.S. + 24^h — R.A. Star.

Whence the Hour Angle will be found to be $17^h\ 20^m\ 54^s$.

Then $\tan l = -\cos. h \cot. d.$

Whence $l = 48°\ 29'$ N.

Ans., $48°\ 29'$ N.

13. *See fig.* 13.

	H.	M.	S.
Time by clock - - -	19	31	56
Error fast - - -		5	27
Sidereal Time - -	19	26	29

From the figure—

$$Q'm = AQQ' - Am.$$

Or

Ship Mean Time = Sidereal Time — R.A.M. Sun.

Whence we obtain an approximate value for Ship M.T., and thus the Greenwich Date for correction of R.A.M. Sun Using the value so obtained, and repeating the process, we obtain G.M.T. $1^h\ 6^m\ 37{\cdot}4^s$, and the error of the chronometer $5^h\ 42^m\ 32{\cdot}6^s$ fast.

Ans., $5^h\ 42^m\ 32{\cdot}6^s$ fast.

14. *See fig.* 14.

First to find the hour angle of the Sun at rising.

$$\text{Cos. } h = -\tan l \tan d.$$

Whence $h = 16^h\ 52^m\ 37^s$.

And Ship M.T. = Ship A.T. + Equation of Time = $16^h\ 57^m\ 24^s$.

And since the Transit of the First Point of Aries took place at $14^h\ 36^m\ 57^s$ Ship M.T., it follows that $2^h\ 20^m\ 27^s$ of Mean Time have elapsed since this transit.

We have therefore only to turn this interval into Sidereal Time to obtain the Sidereal Time of Sunrise.

	H.	M.	S.
Sidereal Time of Sunrise -	2	20	50
Error fast - - - -		8	5
Time shewn by clock - -	2	28	55

Ans., $2^h\ 28^m\ 55^s$.

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15. *See fig.* 15.

From the figure—

$$AQQ' = Am + mQ', \text{ or}$$

Sidereal Time = R.A.M. Sun + Mean Time.

$$= 12^{h.}\ 43^{m.}\ 46{\cdot}4^{s.} + 5^{h.}\ 49^{m.}\ 30^{s.} = 18^{h.}\ 33^{m.}\ 16{\cdot}4^{s.}$$

Ans., $18^{h.}\ 33^{m.}\ 16{\cdot}4^{s.}$

16. *See fig.* 16.

First to find the hour angle of Saturn at rising, we have—

$$\text{Cos. } h = -\tan l \tan d.$$

whence h will be found to be $17^{h.}\ 18^{m.}\ 5^{s.}$.

Proceeding as in previous problems, Ship M.T. = $9^{h.}\ 40^{m.}$ nearly, and the Greenwich Date is $2^{h.}\ 28^{m.}$.

We can now correct R.A. Mean Sun, and R.A. Saturn for this Greenwich Date; then, making use of these corrected elements, we shall find Ship M.T. = $9^{h.}\ 39^{m.}\ 10{\cdot}5^{s.}$

Ans., $9^{h.}\ 39^{m.}\ 10{\cdot}5^{s.}$

17. *See fig.* 17.

First to find the hour angle of the star.

From the figure Q'R = AQQ' − AR, that is,

H.A. Star = Sidereal Time − R.A. Star.

	H.	M.	S.
Time by Sidereal clock - -	10	10	31
Error fast on Greenwich -	2	1	17
Sidereal Time at Greenwich -	8	9	14
Longitude in time - - -	5	18	0
Local Sidereal Time - -	13	27	14
R.A. Star - - - -	10	1	54
H.A. Star - - - -	3	25	20

Having now in the triangle PZX, the side PZ = 39°, PX = 77° 26′ 20″, and the included angle ZPX = $3^{h.}\ 25^{m.}\ 20^{s.}$, we may find the third side ZX.

Ans., 33° 33′ 30″.

18. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 136.

Let x be the number of degrees of longitude which the required meridian lies to the westward of Greenwich.

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The interval elapsing between $7^{h.}\ 53{\cdot}4^{m.}$ May 18th, and $4^{h.}\ 59^{m.}$ May 19th, is $21^{h.}\ 5{\cdot}6^{m.}$

Thus the Moon passes the required meridian $21^{h.}\ 5{\cdot}6^{m.}$ after passing the meridian of Greenwich.

If the meridian were 360° westward of Greenwich, that is, if the two transits took place over the same meridian, an interval of $24^{h.}\ 44{\cdot}4^{m.}$ would elapse.

Thus we have the proportion—

$$\frac{x}{360^{\circ}} = \frac{21^{h.}\ 5{\cdot}6^{m.}}{24^{h.}\ 44{\cdot}4^{m.}}$$

And $x = 306^{\circ}\ 56'$ nearly, *i.e.*, the longitude is 53° 4′ E.

Ans., 53° E. nearly.

19. Since the Moon crossed the meridian of Greenwich at $5^{h.}\ 53{\cdot}6^{m.}$, but for her proper motion she would cross the meridian of 125° W. at $5^{h.}\ 53{\cdot}6^{m.} + 8^{h.}\ 20^{m.}$ or at $14^{h.}\ 13{\cdot}6^{m.}$.

The difference between this value and the actual time of transit, which is $17{\cdot}1^{m.}$, must be the retardation upon solar time, due to this proper motion, for 125° of longitude.

It is required to find x, the retardation for 360° of longitude.

Thus—
$$x = \frac{360}{125}\ 17{\cdot}1^{m.} = 49{\cdot}248^{m.}$$

Ans., at $6^{h.}\ 42{\cdot}8^{m.}$

20. In this case the proper motion is retrograde. The correction to be applied on account of this proper motion must therefore be added instead of being subtracted, as would be the case if the motion were direct.

To find the amount of the correction, let x be its value in minutes.

Then—
$$x = \frac{119}{360}\ 4^{m.} = 1{\cdot}3^{m.} \text{ nearly.}$$

Thus the Ship M.T. of transit is $16^{h.}\ 35{\cdot}9^{m.}$

Ans., at $16^{h.}\ 35{\cdot}9^{m.}$

21. *See fig.* 18.

To find the Ship M.T.

From the figure—

$$Q'm = Q'R - m\,R = Q'R - (\text{R.A.M.S.} - \text{R.A. Star}).$$
$$= \text{Star's H.A.} - \text{R.A.M.S.} + \text{R.A. Star.}$$

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From this formula we obtain an approximate value for the Ship Mean Time, and thence the Greenwich Date, in order to correct the R.A.M. Sun. Making use of the corrected Right Ascension, we find correct Ship Mean Time to be $7^{h.}\ 34^{m.}\ 34^{s.}$

To find the time of the Star's next transit over the meridian. The hour angle is $11^{h.}\ 13^{m.}\ 48^{s.}$ The transit will take place therefore in $46^{m.}\ 12^{s.}$ of Sidereal time, an interval equivalent to $46^{m.}\ 4{\cdot}4^{s.}$ of Mean time.

Ans. { Ship M.T. $7^{h.}\ 34^{m.}\ 34^{s.}$
Transit at $8^{h.}\ 20^{m.}\ 38{\cdot}4^{s.}$

22. *See Jeans' Nautical Astronomy and Navigation*, Part I., p. 175.

Ans., $4^{h.}\ 18^{m.}\ 59^{s.}$

II.—THE SEXTANT.

1. *See Merrifield and Evers' Navigation and Nautical Astronomy*, p. 147.

The degree of accuracy $= \frac{l}{n}$, where l is the value of a division on the limb, and n the number of divisions upon the vernier.

Ans., $15''$.

2. Let x = the number of divisions on the vernier.

Then by the formula used above—

$$20'' = \frac{(15 \times 60)''}{x}, \text{ whence } x = 45.$$

And according to the ordinary arrangement for sextant verniers, the number of divisions on the limb will be one less than 45, or one less than some multiple of 45.

In general the number on the limb is one less than twice the number on the vernier, that is, in the present instance, 89.

Ans., 45 divisions, coinciding with 89 upon the limb.

3. *Ans.*, 90 divisions on the vernier.

4. *Ans.*, $10''$.

5. *Ans.*, 50 divisions, which, according to the ordinary construction of the barometer vernier, will coincide with 51 on the instrument.

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III.—MERIDIAN AND CIRCUM-MERIDIAN ALTITUDES.

1. *See fig.* 19.

Let N W S E represent the horizon, N Z S the celestial meridian, and WQE the equator, so that ZQ represents the latitude.

Let X,X′ be the places of the Sun when upon the meridian, at the summer and winter solstices respectively.

$$\text{Then } ZX = ZQ - XQ = l - d.$$
$$\text{and } ZX' = ZQ + X'Q = l + d.$$
$$\text{Therefore } ZX + ZX' = 2l.$$

But since the two meridian altitudes are complementary, it follows that the meridian zenith distances are complementary.

$$\text{Therefore } ZX + ZX' = 90°.$$
$$\text{Thus } 2l = 90°, \text{ and } l = 45°.$$

Ans., 45°.

2. *See fig.* 20.

By a circumpolar star is meant a star which passes the meridian twice in each twenty-four hours above the horizon.

Since the altitude of the pole, PN, is equal to the latitude, the polar distance of the star must not be greater than the latitude of the place, and the limit of latitude is reached when the latitude is equal to the polar distance.

Ans., between 28° and 90°
Alt. above Pole 62°
„ below „ 6°

3. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 29.

When a heavenly body, such as the Moon, whose declination is subject to rapid change, arrives at the meridian, if its motion in declination causes it to approach the elevated pole, it is obvious that its altitude will increase after passing the meridian, and this increase will continue until the *rate of change* in altitude due to the rotation of the earth is equal to the rate of change of declination caused by the proper motion of the body.

An expression for the time at which a body of variable

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declination attains its maximum declination may be obtained as follows:—

See fig. 21.

Let P be the pole, and Z the zenith.

Let x' be the position of a body of polar distance Px' at a given small hour angle h', and let x be the position which it would occupy at an hour angle h if its declination were invariable.

From x' let fall a perpendicular $x'm$ upon Zx.

Then xm may be considered to represent the change in altitude due to the rotation of the earth during a period xPx', or $h - h'$, which we may conveniently take as one second.

If instead of being fixed, the body's declination be subject to rapid variation, and δ be the change of declination for $h - h'$, one second of time, we shall have to find h, the time when the rate of upward motion due to the body's change of declination, is equal to the downward motion due to the rotation of the earth, so that $xm = \delta$.

For since the hour-angle is inconsiderable, the motion due to change of declination may be considered as taking place upon the celestial meridian, *i.e.*, directly towards the zenith.

The triangle xmx', being very small, may be considered a plane right-angled triangle.

Then we have—

$$\begin{aligned} xm = \delta = xx' \cos. mxx' &= xx' \sin. \mathrm{P}x\mathrm{Z}. \\ &= (h - h') \cos. d \sin. \mathrm{P}x\mathrm{Z}. \\ &= (h - h') \cos. d \cos. l \operatorname{cosec}. \mathrm{Z}x \sin. h. \end{aligned}$$

$$\text{Therefore } \sin. h = \frac{\delta}{(h - h') \cos. l \cos. d \operatorname{cosec}. \mathrm{Z}x.}$$

Or,

$$\text{Sin. } h = \text{sec. } l \text{ sec. } d \text{ sin. } \mathrm{Z}x \frac{\delta}{15} \left\{ \text{since } (h - h') = 1 \text{ sec. (in time)} = 15'' \text{ (in arc).} \right\}$$

And Zx may be taken as the meridian zenith distance, since it can only differ from it by a very small quantity.

And it is obvious that if the motion of the body in declination be such that its distance from the zenith is decreasing, the *maximum* altitude will take place after the body has passed the meridian, but when the distance is increasing, before reaching the meridian.

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4.

	H.	M.	S.
Time by chronometer at second observation -	12	58	16
„ „ first „ -	9	21	41
Elapsed time nearly - - - - -	3	36	35
Lost by chronometer in $3^{h.}$ $36^{m.}$ - - -			1·8
Elapsed time - - - - - -	3	36	36·8

	H.	M.	S.
Ship M.T. at first observation - - -	8	37	46
Elapsed time - - - - - -	3	36	36·8
Ship M.T. at second observation - - -	12	14	22·8
Equation of Time - - - -		7	13
Ship A.T. at second observation - - -	12	21	35·8
Run to the eastward in time - - -		2	2
Ship A.T. corrected for Run - - -	12	23	37·8

Ans., $0^{h.}$ $23^{m.}$ $38^{s.}$

5. (*a*) *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 56.

By formula:—

Error in lat. = cos. lat. sin. azimuth. error in hour angle.

Ans., 5′ 48″.

(*b*) *Ans.*, the errors are as 4·95 to 1 nearly.

6. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 58.

(*a*) *Ans.*, from $6^{h.}$ $19^{m.}$ to $6^{h.}$ $59^{m.}$.

(*b*) Assuming that the Moon's declination always lies between 0° and 28° 37′ N. or S., the lowest latitude in which the Moon can pass the meridian below the Pole above the horizon will be 61° 23′.

Ans., from 61° 23′ to 90°.

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IV.—LATITUDE BY DOUBLE ALTITUDE.

1. *See Jeans' Nautical Astronomy and Navigation*, Part I., p. 143.

By formula:—

Correction for Run = distance run × cos. angle between the direction of the Run and the bearing of the Sun at the first observation.

If this angle be 90°, since cos. 90° = 0, the correction must be zero.

The condition, therefore, is that the course of the Ship shall be at right angles to the bearing of the Sun at the first observation.

2. Combining the courses and distances made good by the ship, by means of the Traverse Table, we obtain a single course and distance of S. 67° E., 13′.

By formula:—

Correction = 13′ × cos. angle between Sun's bearing and Run.

= 13′ cos. 14° 34′ = 12′ 36″.

Ans., 12′ 36″, additive to first altitude.

3 The two courses when combined will give a single course and distance of S. 79° E., 5·5′.

If the altitude required be that for the first place of observation, this must be reversed, and becomes N. 79° W., 5·5′.

And the bearing of the Sun was N.N.W. at the second observation.

Thus—

Correction = 5·5′ cos. 56° 30′.

= 3′ nearly.

Ans., + 3′ nearly.

4. *Ans.*, + 1′ 8″.

5. *See fig.* 22.

Let S be the position of the Sun, Z the zenith at the first observation, Z′ the zenith at the second observation, so that ZZ′ is the arc moved through by the zenith in the celestial concave, corresponding to the path of the ship on the surface of the earth.

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Then—

$$\text{Cos. } Z'ZX = \frac{ZX}{ZZ'} = \frac{\text{correction for Run}}{\text{distance run by ship}} = \frac{13{\cdot}417'}{31{\cdot}5'}.$$

Whence $Z'ZX = 64° 47'$.

Thus the angle SZZ' is 115° 13′, and the bearing of the Sun is N. 53° 32′ E.

Ans., N. 53° 32′ E.

6. N. 4° 46′ E.

7. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 73.

See fig. 23, *drawn on plane of horizon.*

Since y was taken first, let Pr be the circle of declination passing through y, at the moment when x was observed.

The polar angle may be defined to be the angle at the pole between the circles of declination passing through the bodies at the moments when they were actually taken.

Thus in the present instance R′PR, measured by the arc R′R, is the polar angle.

From the figure—

$$R'R = R'r + rR.$$
$$= R'A + Ar + rR.$$
$$= 24^{h.} - \text{RA of } x + \text{RA of } y + \text{elapsed sidereal time.}$$

	H.	M.	S.
$24^{h.}$ − RA of x - -	1	30	4
RA of y - - - -	1	15	21
Elapsed sidereal time -	1	52	39·5
Polar angle - - -	4	38	4·5

Ans., $4^{h.}\ 38^{m.}\ 4{\cdot}5^{s.}$

8. In the case of a single star observed twice, the polar angle will be the elapsed time expressed in sidereal time.

Ans., $3^{h.}\ 50^{m.}\ 5{\cdot}7^{s.}$

9. *See fig.* 24, *drawn on plane of equator.*

Let PR′, Pr be circles of declination passing through α Arietis and Altair respectively at the same instant of time.

Then rR' = R.A. α Arietis + $24^{h.}$ − R.A. Altair.

$= 6^{h.}\ 15^{m.}\ 42^{s.}$

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Since this quantity exceeds the polar angle given, viz., $5^{h.}\ 58^{m.}\ 11^{s.}$, it is evident that Altair must have been taken earlier.

Let PR be the circle of declination passing through Altair when observed, so that RR′ measures the polar angle.

Then—

$$rR = rR' - RR' = 6^{h.}\ 15^{m.}\ 42^{s.} - 5^{h.}\ 58^{m.}\ 11^{s.}$$
$$= 0^{h.}\ 17^{m.}\ 31^{s.} \text{ in sidereal time.}$$

Ans., Altair was taken first; interval between observations $17^{m.}\ 28^{s.}$ by chronometer.

10. *Ans.*, Markab; $24^{m.}\ 21^{s.}$ earlier than Vega.

11. *See fig.* 25.

Let RPr represent the sidereal interval of time between the two observations.

Since the planet's Right Ascension is decreasing, it will be situated, after the given interval, upon a circle of declination nearer to the meridian than Pr, as upon PR′.

And since the polar angle is the angle between the circles of declination passing through the body at the times when it was actually observed, R′R measures the polar angle.

From the figure—

$RR' = Rr + rR'$.

= interval in sidereal time + decrease in R.A. of planet.

Ans., $2^{h.}\ 49^{m.}\ 8^{s.}$

12. (*a*) *Ans.*, $4^{h.}\ 0^{m.}\ 42^{s.}$

(*b*) *Ans.*, $3^{h.}\ 57^{m.}\ 40^{s.}$

V.—SUMNER'S METHOD OF FINDING LATITUDE.

1. *See Merrifield and Evers' Navigation and Nautical Astronomy*, p. 310.

2. *See Merrifield and Evers' Navigation and Nautical Astronomy*, p. 318.

3. The line of bearing of the body is at right angles to the "line of position" obtained from its altitude.

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We have therefore to calculate the bearing of a point in lat. 35° 30′ N., long. 14° 24′ W., from one in 35° 50′ N., long. 14° 5′ W.

The line of bearing of the body will be 90° from the line of position so found.

Ans., S. 52° E.

4. (*a*) *Ans.*, S. 22° E.

(*b*) *Ans.*, S. 66° E.

5. *Ans.*, 27° 13′ W.

6. *See fig.* 26.

Let *ab* be the "line of position" given by the observed altitude, at right angles to the bearing of the Sun at the time.

If we lay off *ac* 12 miles in an E.N.E. direction, and through *c* draw *cd* parallel to *ab*, *cd* will be the line upon which the ship is actually situated.

Let H be the position of the known headland, the bearing of which, corrected for variation and deviation, is S.E.$\frac{1}{2}$E.

Then if we lay off a line from H, N.W.$\frac{1}{2}$W., the point of intersection of this line with *cd* will give the place of the ship.

VI.—LONGITUDE BY CHRONOMETER.

1. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 43.

Ans., N. 35° 33′ E. or W.

2. *Ans.*, 40° 5′.

3. *Ans.*, As 1 to 1·765 nearly.

6. Let e be the error for S. 15° E., e_1 that for S. 75° E.

Then

$$\frac{e}{e_1} = \frac{\frac{1}{\text{sin. } 15^\circ}}{\frac{1}{\text{sin. } 75^\circ}} = \frac{\text{sin. } 75^\circ}{\text{sin. } 15^\circ} = \text{tan. } 75^\circ = 2 + \sqrt{3}.$$

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VII.—LONGITUDE BY LUNAR.

1. The correction to be applied to the apparent altitude of the Sun or Moon, in order to obtain the true altitude, consists of the algebraical sum of the two corrections, for Parallax and Refraction respectively. Of these the correction for Parallax is additive, while that for Refraction is subtractive. For the Moon, on account of its comparative proximity to the earth, the correction for Parallax is greater than the correction for Refraction, and the algebraical sum of the two is therefore positive. In the case of the Sun, since, by reason of its distance from the earth, the effect of Parallax is insignificant, the correction for Refraction is the greater of the two, and the correction, due to their joint effect, is therefore subtractive.

2. *See Jeans' Nautical Astronomy and Navigation,* Part II., p. 83.

The accuracy with which the longitude may be determined in a Lunar observation depends to a great extent upon the rapidity with which the Moon is approaching or leaving a heavenly body. Now the proportional logarithm given in the *Nautical Almanac* is simply the logarithm of 10,800 diminished by the number of seconds in the difference of distance for the particular three-hour interval within which the time of observation falls. Consequently, the greater the difference of distance the smaller will be the proportional logarithm. We may therefore consider that the body which has a comparatively small proportional logarithm is more favourably circumstanced than one which has a larger logarithm.

3. *See fig.* 27.

In the Lunar observation we obtain by means of a sextant ZA, ZB the true zenith distances of the Sun or Star, and Moon respectively. And from A′B′, the apparent distance of the two bodies, we obtain AB the true distance.

Now AB corresponds to what is known as Arc I. in the ordinary method of finding latitude by Double Altitude. (*See Jeans' Navigation and Nautical Astronomy,* Part II., p. 67). Starting from that point, therefore, we may proceed to find PZ, the co-latitude, as there explained.

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4. *See Harbord's Glossary of Navigation*, under "Lunar Distance Bodies."

Since the value of the observation depends upon the rapidity with which the moon is altering her distance from the body observed, the bodies selected for tabulation in the *Nautical Almanac* must all lie within a certain distance of the moon's path. All the lunar distance bodies, therefore, are selected from those which lie within a belt extending to about 30° on each side of the equator.

5. *See Jeans' Navigation and Nautical Astronomy*, Part II., p. 87.

Let t = time elapsed from $9^{h.}\ 0^{m.}$

d = diff. of dist. for 3 hours = 1° 31′ 34″.

x = change required in $46^{m.}\ 50^{s.}$.

By formula,

$$\text{Prop. log. } t = \text{prop. log. } x - \text{prop. log. } d.$$

Therefore,

$$\text{Prop. log. } x = \text{prop. log. } t + \text{prop. log. } d.$$

Ans., 86° 11′ 39″.

6. From the given altitude of the Sun's centre we deduce the hour angle of the Sun, $5^{h.}\ 31^{m.}\ 51^{s.}$ By means of R.A. Sun and R.A. Moon, we obtain the hour angle of the moon, which is found to be $2^{h.}\ 18^{m.}\ 1^{s.}$; whence, with the co-latitude and polar distance of the moon, we may obtain the altitude of Moon's centre.

Ans., 56° 19′ nearly.

VIII.—VARIATION AND DEVIATION OF COMPASS.

1. *See fig.* 28.

Let ON be the direction of correct magnetic north, which will therefore be that of the needle of the standard compass.

Lay off an angle NOS of 74°.

Then OS will be the direction of the ship's head.

And since the bearing of the ship's head by the azimuth compass is N. 77° E., by measuring back an angle of 77° from OS, we arrive at the direction of the needle of the azimuth compass, ON′, which is therefore 3° to the westward of ON.

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And the bearing of the peak by the azimuth compass is N. 50° W.

The peak, therefore, lies on a line of bearing OP, which is 50° to the left of ON′.

And NOP = NON′ + N′OP = 3° + 50° = 53°.

Ans., N. 53° W.

2. *See fig.* 29.

Since the ship found herself too much to the southward, it is evident that she must, in finding compass course from true course, have allowed to the right instead of the left; so that the variation, which was easterly, was applied as if westerly.

Let S, S′ be the two positions of the ship, and P that of the port, so that SP = 120, PS′ = 50, S′S = 100.

Solving the triangle SPS′, we find that the angle S is 24° 9′.

And this angle is manifestly equal to twice the amount of variation.

Ans., 12° 4′ E.

3. *See Jeans' Navigation and Nautical Astronomy*, Part II., p. 106.

In high latitudes the small circle which represents the Sun's diurnal path will be so nearly parallel to the horizon, that it becomes difficult to fix with certainty upon the exact moment when the centre is in the horizon. An error of several degrees in the observed amplitude may therefore easily result.

4. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 112. *See fig.* 30.

Let X be the true place of the sun, O that of the object projected upon the celestial concave, so that OX is the observed distance.

In the triangle PZX, we have PZ = 55°, PX = 80°, ZX = 52°, whence PZX will be found to be 106° 8′ 45″.

And in the triangle OZX, ZO = 90°, ZX = 52°, and OX = 96°.

The angle OZX will be found to be 97° 37′ 15″.

And PZO the true bearing of the Sun = 360° − PZX − OZX = 156° 14′.

Thus the true bearing is N. 156° 14′ W., and compass bearing N. 160° W., so that the compass error is 3° 46′ E.

Ans., 3° 46′ E.

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Note.—The observed distance OX is measured from the object to the apparent place of the Sun, while the true bearing obtained by calculation is that of the true place of the Sun. In a practical problem of this nature, however, it is unnecessary to take this distinction into account, and the two places are supposed to coincide.

5. *Ans.*, N. 15° 9′ E.

6. *Ans.*, N. 17° 1′ W.

7. *See Raper's Practice of Navigation*, Art. 446.

8. The amplitude of the Moon is unsuitable for observation, because on account of her proximity to the earth the parallax is so great, that to an observer on the earth's surface the Moon does not appear to rise until after the proper time. The joint effect of parallax and refraction is to depress the centre about half a degree, at the instant when it ought actually to appear in the horizon.

In the case of the Sun the effect is to raise the centre by about the same amount.

It is therefore pointed out by *Raper* in his *Practice of Navigation*, Art. 886, that the correction to be taken from Table 59A, calculated for the Sun, must be applied in the contrary direction when the Moon is selected for observation.

9. We have by the ordinary formula for finding the true amplitude, A,

Sin. A = sin. d sec. l, where d = declination of Sun, and l the latitude of the place.

Since A = 2d this becomes—

$$\text{Sin. } 2d = \text{sin. } d \text{ sec. } l.$$

$$\text{And cos. } d = \frac{1}{2} \text{ sec. } l.$$

Whence $d = 12° 42'$.

Making use of this value for the declination we find the compass error 8° 24′ W., so that, allowing for the deviation 5° 30′ W., the variation must be 2° 54′ W.

Ans., 2° 54′ W.

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IX.—MISCELLANEOUS PROBLEMS.

1. The hour angle of Regulus will be found to be $21^{h.}\ 9^{m.}\ 48^{s.}$. Adding the Right Ascension of the Star, and subtracting that of the Mean Sun, we shall find the Ship Mean Time to be $4^{h.}\ 5^{m.}\ 37^{s.}$ A.M.

Ans., at $4^{h.}\ 5^{m.}\ 37^{s.}$ A.M.

2. *See fig.* 31.

Let S be the position of the star, AQ, AQ' arcs of the ecliptic and equator respectively.

Drop perpendiculars SM, SN.

In the triangles SAM, SAN, AM, the longitude, is equal to SN the declination; AS is common, and the angles at M, N are each right angles.

Therefore the two triangles are equal in all respects.

Therefore the side SM is equal to the side AN, that is, the latitude is equal to the Right Ascension.

3. *See fig.* 32.

Let A be the position of Pollux at rising, B that of Rigel.

Let d_1 be the declination of Pollux, d_2 that of Rigel, and let ZPA = h_1, ZPB = h_2.

In the triangles PZA, PZB, since ZA, ZB are each a quadrant,

$$\text{Cos. } h_1 = -\tan l \tan d_1$$
$$\text{Cos. } h_2 = \tan l \tan d_2.$$

$$\text{Therefore } \frac{\text{Cos. } h_2}{\text{Cos. } h_1} = -\frac{\tan d_2}{\tan d_1}$$

$$\therefore \frac{\text{Cos. } h_2 - \text{cos. } h_1}{\text{Cos. } h_2 + \text{cos. } h_1} = -\frac{\tan d_1 + \tan d_2}{\tan d_1 - \tan d_2} = -\frac{\text{sin. } (d_1 + d_2)}{\text{sin. } (d_1 - d_2)}$$

$$\therefore \text{Tan } \frac{h_1 + h_2}{2} = -\frac{\text{sin. } (d + d_2)}{\text{sin. } (d_1 - d_2)} \text{cot. } \frac{h_1 - h_2}{2}$$

Now $h_1 - h_2$, being the difference of the stars in Right Ascension, is $2^{h.}\ 30^{m.}$, so that we can now determine $\frac{h_1 + h_2}{2}$, which will be found to be $6^{h.}\ 44^{m.}\ 0^{s.}$.

Thus h_1 is $7^{h.}\ 59^{m.}$.

Then l may be determined from the relation—

$$\text{Tan } l = -\cos h_1 \text{ cot. } d_1,$$

Whence the latitude will be found to be 42° 39′ N.

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In the above example we have deduced the latitude from a special formula, whereby the labour of computation is very much reduced. If, however, we examine the data of the question, we find that we have given the zenith distances of two stars, of which the declinations and right ascensions are also known. The problem therefore may be solved as a simple case of finding latitude by double altitude of different stars observed at the same time.

Worked in this manner we shall first find the arc AB, which is 51° 31′, and the angle PAB, 129° 41′ 30″.

And the angle ZAB is a right angle, so that PAZ is 39° 41′ 30″.

Then in the triangle APZ, having given PA, the polar distance of Pollux, the angle A, and ZA a quadrant, we may find PZ.

Ans., 42° 39′ N.

4. *See fig.* 33.

The latitude may be either north or south; in the present case let us assume that it is north. Then since the Sun rises later than 6 A.M., the declination must be south.

From the quadrantal triangle PZX we have—

$$\text{Cos. } h = \tan l \tan d.$$

And from the figure, if X′ be the place of the Sun at noon—

$$\text{QX}', = 2\text{ZQ}, \text{ or } d = 2l.$$

Substituting tan. $2l$ for tan. d we have—

$$\text{Cos. } h = \frac{2 \tan^2 l}{1 - \tan^2 l}$$

Therefore, adding unity to each side of the equation—

$$1 + \text{cos. } h = \frac{1 + \tan^2 l}{1 - \tan^2 l}$$

Or

$$\text{Sec. } 2l = 2 \text{ cos.}^2 \frac{h}{2}.$$

Whence the latitude will be found to be 10° 5′ N.

Ans., 10° 5′ N. or S.

5. *See fig.* 33.

As in the preceding example, if h be the distance of the Sun in hour angle from the meridian at rising.

$$\text{Cos. } h = \pm \tan l \tan d.$$

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And if the declination be considered constant, h will be one-half of the length of the day.

If $l = 0$, cos. $h = 0$, and $h = 6$ hours, so that the day at the equator is always 12 hours long, whatever the declination.

If $d = 90° - l$, cos. h = tan l cot. $l = 1$, and $h = 0$, so that when the Sun is in the horizon it is also upon the meridian.

If d is greater than $(90° - l)$, since cos. $h = \frac{\tan d}{\cot. l} = \frac{\tan d}{\tan (90° - l)}$, and tan d must be greater than tan $(90° - l)$, the equation gives a value for cos. h which is greater than unity, and h therefore becomes impossible.

So long, therefore, as the declination is greater than the complement of the latitude, the Sun will remain constantly above the horizon.

6. *See fig.* 34.

Let X be the position of the heavenly body on reaching the prime vertical west of meridian.

In the right-angled triangle PXZ we have—

$$\text{Cos. } h = \text{cot. } l \text{ tan } d$$

When the declination is zero, cos. $h = 0$, so that $h = 6$ hours.

When the declination is greater than the latitude, since cos. $h = \frac{\tan d}{\tan l}$, the equation becomes impossible.

When the declination and latitude are equal, cos. $h = 1$, and $h = 0$; the body will therefore be upon the meridian and prime vertical at the same instant, namely, when it is in the zenith.

7. *See fig* 35.

Let h_1 be the hour angle of A, the star of northern declination, h_2 that of B, the star of southern declination.

As in previous problems we have—

$$\text{Cos. } h_1 = -\text{ tan } l \text{ tan } d.$$

$$\text{Cos. } h_2 = \text{tan } l \text{ tan } d.$$

Therefore cos. $h_1 = -$ cos. h_2, and $h_1 + h_2 = 12$ hours.

And $h_1 - h_2 =$ APB = the difference of the two stars in right ascension.

This difference must be equal to the interval in sidereal

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time between the transits of the two stars over the meridian $= 1^{h.}\ 59^{m.}\ 40^{s.} + 19{\cdot}65^{s.} = 2^{h.}\ 0^{m.}\ 0^{s.}$, very nearly.

Thus $h_1 = 7^{h.}$, $h_2 = 5^{h.}$

And tan l = cos. $5^{h.}$ cot. 15°.

Ans., 44° 0′ N.

8. *Ans.*, at $7^{h.}\ 20^{m.}$ A.M.; amplitude E. 30° 28′ S.

9. *See fig.* 36.

Let x be the position of the star when upon the meridian below Pole, X that of the star when its azimuth is greatest.

Since in the triangle PXZ,

Sin. Z $= \frac{\text{sin. } p}{\text{sin. } c}$ sin. X, and p, c are constant, the azimuth is greatest when X is a right angle.

Thus in the right-angled triangle PXZ we have—

Sin. PX = sin. PZ sin. Z.

Or, Cos. d = cos. l sin. a, where a is the inferior meridian altitude.

But $a = l - p = l - 90° + d = l - 15°$.

Therefore

$$\text{Cos. } l \text{ sin. } (l - 15°) = \text{cos. } 75°.$$

$$\therefore\ \frac{1}{2}\left\{\text{sin. } (2l - 15°) - \text{sin. } 15°\right\} = \text{cos. } 75°.$$

$$\therefore\ \text{Sin. } (2l - 15°) = 2 \text{ cos. } 75° + \text{sin. } 15° = 3 \text{ cos. } 75°.$$

Ans., 32° 58′ N.

10. Let c = co-latitude, p = polar distance of star.

Since $l - p = 30°$.

$90° - c - p = 30°$.

$\therefore\ c + p = 60°$.

Again, in the triangle PXZ. (*See fig.* 36).

$$\frac{\text{Sin. } c}{\text{Sin. } p} = \frac{2}{1}$$

$$\therefore\ \frac{\text{Sin. } c - \text{Sin. } p}{\text{Sin. } c + \text{Sin. } p} = \frac{1}{3}$$

$$\therefore\ \text{Tan } \frac{c - p}{2} = \frac{1}{3} \tan \frac{c + p}{2} = \frac{1}{3} \tan 30°.$$

Whence $\frac{c - p}{2}$ will be found to be 10° 53′ 30″.

Ans., lat. 49° 7′ N.; dec. 70° 53′ N.

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11. *See fig.* 37.

Let x, y be the positions of the Star at the two observations.

Drop a perpendicular Pv from P upon the vertical circle passing through xy, and join Px, Py.

The right-angled triangle Pvy is equal in all respects to Pvx.

Therefore vx, vy each $= 3° 15'$.

In the triangle Pvy we have—

$$\text{Cos. P}yv = \tan d \tan 3° 15' = \tan 78° 30' \tan 3° 15'.$$

Whence the angle Pyv will be found to be $73° 47' 30''$, and the angle PyZ is therefore $106° 12' 30''$.

Then in the triangle PyZ we have—

$$\text{Cos. } l = \text{cos. } d \text{ sin. P}y\text{Z cosec. } 22° 30'.$$

Ans., $59° 59'$ N.

12. As in previous problems we have for the polar distance $45° - a$, where a is the inferior meridian altitude.

Let the maximum azimuth be A.

$$\text{Sin. A} = \text{sin. } 2a = \frac{\text{sin. } p}{\text{sin. } 45°} = \frac{\text{sin. } (45° - a)}{\text{sin. } 45°} = \text{cos. } a - \text{sin. } a.$$

Therefore

$$\text{Sin.}^2 2a = 1 - \text{sin. } 2a.$$

Solving this quadratic equation we obtain—

$$\text{Sin. } 2a = \frac{\sqrt{5} - 1}{2}.$$

Whence $a = 19° 5'$.

Ans., $64° 5'$ N.

13. *See fig.* 38.

Let QR_2Q' be the equator, and P the Pole.

Let $Q'PR_1$, $Q'PR_2$ be the distances from the meridian of the Apparent Sun at rising and setting respectively, and let m_1, m_2 be the corresponding positions of the Mean Sun, which is obviously behind the real Sun in time.

Then R_1m_1 or R_2m_2 is the equation of time.

Regarding the declination as constant for the day,

$$Q'R_1 = Q'R_2.$$

$$\therefore Q'm_1 - R_1m_1 = Q'm_2 + m_2R_2.$$

$$\therefore R_1m_1 + m_2R_2 = Q'm_1 - Q'm_2.$$

Thus the equation of time is one-half the difference between

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the distances of the Mean Sun from the meridian at the rising and setting of the real Sun.

Ans., 16 minutes, subtractive from Apparent Time.

14. *See fig.* 39.

Let A be the position of the Sun at rising, h_1 its distance in hour angle from the meridian.

Let B be the position of the Star, and h_2 its distance from the meridian.

Then $h_1 - h_2$ = the difference of the two bodies in Right Ascension, $= 2^{h.}\ 15^{m.}\ 9^{s.}$.

In the triangle ZPA—

$$\text{Cos. } h_1 = -\tan l \tan 21°.$$

In the triangle ZPB—

$$\text{Cos. } h_2 = \tan l \tan 15°\ 10'.$$

Therefore

$$\frac{\text{Cos. } h_2 - \text{cos. } h_1}{\text{Cos. } h_2 + \text{cos. } h_1} = \frac{\tan 15°\ 10' + \tan 21°}{\tan 15°\ 10' - \tan 21°} = -\frac{\text{sin. } 36°\ 10'}{\text{sin. } 5°\ 50'}$$

$$\therefore \tan \frac{h_1 + h_2}{2} = -\frac{\text{sin. } 36°\ 10'}{\text{sin. } 5°\ 50'} \text{ cot. } 1^{h.}\ 7^{m.}\ 35^{s.}.$$

Whence $h_1 + h_2$ may be obtained, and $h_1 - h_2$ is already known, so that the values of h_1, h_2 may be found, and thence that of l.

Ans., 41° 33′ N.

NOTE.—See the note appended to the solution of Question 3 for an alternative method of solving this problem.

15. *See fig.* 40.

Let A be the position of Fomalhaut when taken, B that of α Pegasi.

Since the elapsed interval was equal to the difference of the stars in Right Ascension, it is obvious that they were observed upon the same hour circle.

Join ZA, ZB.

In the triangle AZB we have given the three sides, viz., ZA, ZB the two zenith distances, and AB the sum of the declinations, so that we may find the angle A.

This will be found to be 125° 43′ 45″.

Therefore the angle PAZ = 180° − A = 54° 16′ 15″.

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And in the triangle PZA, having now the two sides PA, ZA, and the included angle A, we may find the third side PZ, the co-latitude, and thence the latitude.

Ans., 45° 26′ S.

16. *Ans.*, 10° 44′ N.

17. *See fig.* 41.

When the declination of a heavenly body is less than the latitude of a place on the earth, at that place the body will have its greatest azimuth at rising, and this azimuth will continually decrease until the body reaches the meridian.

When, however, the declination is greater than the latitude of the place, and of the same name, this is not so, and the azimuth will increase for some time after rising.

Thus the shadow of an object will move for a certain time away from the meridian, until the azimuth attains its maximum value, when, after remaining stationary for a time, it will commence moving in the contrary direction.

And, as in the solution of Question 9, it may be easily shown that the maximum value is attained when the angle between the circles of declination and of altitude passing through the body is a right angle.

Let A be the place of the Sun at rising.

Let X be the place on attaining its maximum azimuth.

In the quadrantal-triangle ZPA—

$$\text{Cos. } 6^{h.}\ 7^{m.}\ 18^{s.} = -\tan l \tan d.$$

In the right-angled triangle ZPX—

$$\text{Cos. } 5^{h.}\ 4^{m.}\ 22^{s.} = \tan l \cot. d.$$

Thus $\tan^2 l = -\cos. 6^{h.}\ 7^{m.}\ 18^{s.} \cos. 5^{h.}\ 4^{m.}\ 22^{s.}$

Ans., 5° N. or S.

18. The hour angle of Pollux will be found to be $3^{h.}\ 31^{m.}\ 32^{s.}$, which is therefore the interval of sidereal time that has elapsed since the meridian passage. Expressed in mean solar time this interval is $3^{h.}\ 30^{m.}\ 57^{s.}$, which has therefore to be added to the time of meridian passage, viz., 5 hours.

Ans., at $8^{h.}\ 30^{m.}\ 57^{s.}$

19. *See fig.* 42.

Let A, B be the places of the Sun at the two observations respectively.

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Join PA, PB, and drop a perpendicular PV upon AB. This will bisect AB.

In the triangle PVZ, V is a right angle, and the angle Z and side PZ are known. Thus ZV may be determined.

In the triangle PBZ, PB, PZ are known, as also is the angle Z. Thus ZB, PBZ may be determined.

In the triangle PBV, V is a right angle, and the angle B, which is the supplement of PBZ, is known; PB also is known. Therefore BV may be determined.

Then $ZA = ZV + AV = ZV + BV$.

Ans., 9° 59′ and 68° 30′.

20. *See Jeans' Nautical Astronomy and Navigation*, Part II., p. 132.

The distance will evidently be the amount of the dip for 30 feet added to that for 20 feet.

Ans., 9·8 miles nearly.

21. *See fig.* 43.

Let S be the vessel at anchor, A the first place of the ship in motion, B the second place.

In the triangle ABS—

AS is the sum of the amounts of dip for 26 feet and 18 feet, BS is the amount of dip for 26 feet, and the angle S is 45°; we have to find AB and the angle SAB.

Ans., N. 32° 7′ E. 8 knots nearly.

22. *See fig.* 44.

Let t_1 be the distance from the observer to the horizon, t_2 the distance of the object from the horizon.

Then $t_1 + t_2$ may be taken as the distance that the object is visible.

Let r be the radius of the earth.

By Euclid III., 36,—

$t_1^2 = (2r + h)\, h = 2rh + h^2 = 2rh$ nearly (since h is very small compared with r).

$\therefore\ t_1 = \sqrt{2rh}$ nearly.

Similarly

$$t_2 = \sqrt{2rh'} \text{ nearly.}$$

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Therefore

$$t_1 + t_2 = \sqrt{2r}\,(\sqrt{h} + \sqrt{h'}).$$

In the example

$$11{\cdot}7 = \sqrt{2r}\left(\frac{\sqrt{36} + \sqrt{25}}{\sqrt{6080}}\right)$$

Ans., 7920 statute miles.

23. *See fig.* 45.

In the triangle PZx we know the three sides, and can therefore find the angle PZx.

Then in the triangle PZy, having PZ, Py, and the angle PZy, which is the supplement of PZx, we can find the third side Zy.

Ans., 34° 12′.

24. *See fig.* 46.

Let Eq represent an arc of the equator, Ec an arc of the ecliptic.

Let SR be an arc of the circle of declination passing through the Sun.

In the right-angled triangle SAR—

$$\text{Sin. RA} = \text{cot. } \omega \text{ tan } d.$$

But, by hypothesis, RA = (90° − d).

Therefore $\sin.(90° - d) = \frac{7}{3}\tan d.$

$$\therefore \quad 3\cos.^2 d = 7 \sin. d.$$

Ans., d = 21° 43′ N.

25. The meridian altitude must be 45°. The latitude is therefore 34° 30′ N.

To find a, the altitude in the afternoon at the required time, we have—

$$\text{Cot. } a = 3, \text{ and L cot. } a = 10{\cdot}477121.$$

This gives 18° 26′ for the altitude.

Knowing now the altitude and declination of the Sun, and the latitude of the place, we have three sides given in a spherical triangle, and can find the required hour angle.

Ans., at $3^{h.}$ $55^{m.}$ $19^{s.}$ P.M.

26. *See fig.* 47.

Let AB be the stick, the length of which is supposed to be h.

Then h = AC tan 29° 47′ = 2·037 feet.

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To find the condition that the shadow may be longest as the stick rotates round A.

Let θ be the angle that the stick makes with the Sun's rays, which may be considered parallel, and let a represent the Sun's altitude, and l the greatest length of the shadow, AE.

Then in the triangle ADE, $l = \frac{h \text{ sin. } \theta}{\text{sin. } a}$, and h, a being constant, it is evident that l will be greatest when sin. θ is greatest, that is, when $\theta = 90°$.

Thus $l = h$ cosec. a.

Ans., 4·102 feet.

27. *See fig.* 48.

From the data of the question we obtain the zenith distance of the Sun at a certain time in the forenoon, and the zenith distance again when the Sun is upon the same vertical circle in the afternoon.

It is obvious that the Sun must be upon the same vertical circle in the two cases, because the shadow of one stick can only fall at the foot of the other when the two sticks are in the same vertical plane.

If a, β be the zenith distances in the forenoon and afternoon respectively, we have—

$$\text{Tan } a = \frac{18}{4{\cdot}25}, \text{ tan } \beta = \frac{18}{6{\cdot}5}.$$

Whence a, $\beta = 76° 43'$ and $70° 8' 45''$ respectively.

Let Z = the angle PZA.

From the triangles PZA, PZB we have—

$$\text{Cos. Z} = \frac{\text{cos. } p - \text{cos. } c \text{ cos. } a}{\text{sin. } c \text{ sin. } a} = \frac{\text{cos. } c \text{ cos. } \beta - \text{cos. } p}{\text{sin. } c \text{ sin. } \beta}.$$

Therefore

$$\text{Cos. } p \text{ sin. } \beta - \text{cos. } c \text{ cos. } a \text{ sin. } \beta = \text{cos. } c \text{ sin. } a \text{ cos. } \beta - \text{cos. } p \text{ sin. } a.$$

$$\therefore \text{ Cos. } c \text{ sin. } (a + \beta) = \text{cos. } p \text{ (sin. } a + \text{sin. } \beta).$$

$$\therefore \text{ Cos. } c = \frac{\text{cos. } p \text{ cos. } \frac{a - \beta}{2}}{\text{cos. } \frac{a + \beta}{2}}.$$

Ans., 64° 59′ N.

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28. *See fig.* 49.

Let AB be the base of a wall inclined at an angle θ to the horizon, and let AC, CE be the limits of its shadow.

From C let fall a perpendicular, CD, upon AB.

Then if h be the height of the wall, and a the altitude of the Sun—

$$\text{AC} = h \text{ cot. } a, \text{ and CD} = h \text{ cot. } a \text{ sin. } \theta.$$

Similarly for a wall of height h' at right angles to the former one, which is consequently inclined to the meridian at an angle $90° - \theta$, we shall have breadth of shadow $= h'$ cot. a sin. $(90° - \theta)$ $= h'$ cot. a cos. θ.

From these two equations we may obtain a and θ.

Ans., altitude 30° 42′, walls inclined to meridian 46° 46′, and 43° 14′ respectively.

29. Let h = the distance in hour angle from the meridian of the Sun at rising $= 12^{h.} - 4^{h.}\ 49^{m.} = 7^{h.}\ 11^{m.}$

Then

$$\text{Cos. } h = -\tan l \tan d = -\tan d \tan 2d = -\frac{2 \tan^2 d}{1 - \tan^2 d}$$

Therefore

$$\frac{1 - \tan^2 d}{2 \tan^2 d} = -\frac{1}{\cos. h}$$

Adding unity to each side of the equation we have—

$$\frac{1 + \tan^2 d}{2 \tan^2 d} = -\frac{1 - \cos. h}{\cos. h}$$

Or

$$\text{Cosec.}^2 d = -4 \text{ sec. } h \text{ sin.}^2 \frac{h}{2}$$

Ans., $d = 19° 59′$ N. or S., $l = 39° 58′$ N. or S.

30. Let a = the altitude of Sun at $4^{h.}$ P.M. $= 31° 45′$.

h = hour angle at setting - - $= 6^{h.}\ 45^{m.}$

Then

$$\text{Cos. } 4^{h.} = \frac{1}{2} = \frac{\text{sin. } a - \text{sin. } l \text{ sin. } d}{\text{cos. } l \text{ cos. } d}$$

$\therefore$ Cos. l cos. $d = 2$ sin. $a - 2$ sin. l sin. d. (1)

But since cos. $h = -$ tan l tan d—

$\therefore$ 2 cos. l cos. d cos. $h = -2$ sin. l sin. d. (2)

Subtracting equation (2) from (1)—

$$\text{Cos. } l \text{ cos. } d \ (1 - 2 \text{ cos. } h) = 2 \text{ sin. } a.$$

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Let $\tan^2\theta = -2\cos. h$, so that $\theta = 31° 59' 30''$.

Then

$$\text{Cos. } l \cos. d\,(1 + \tan^2\theta) = 2 \sin. a.$$

$$\therefore \text{ Cos. } l \cos. d = 2 \sin. a \cos.^2\theta. \qquad (3)$$

Making use in equation (3) of the value of θ found above, we shall find that cos. l cos. d = ·75703.

And, from (2), sin. l sin. $d = -$ cos. l cos. d cos. h = ·14768.

Knowing now the value of cos. l cos. d and sin. l sin. d, we may find the value of cos. $(l + d)$, their difference, and cos. $(l - d)$, their sum, and thence of l and d.

Ans., lat. 38° 50′ N. or S.; dec. 13° 37′ N. or S.

31. By the formula for amplitude—

$$\text{Sin. } 30° = \frac{\text{sin. } d}{\text{cos. } l}$$

Therefore cos. l = cosec. 30° sin. 15° = $2\,\dfrac{\sqrt{3}-1}{2\sqrt{2}} = \dfrac{\sqrt{3}-1}{\sqrt{2}}$.

32. At the equinoxes the inclination of the ecliptic to the equator is approximately 23° 28′.

As the Sun increases its longitude, its path will be inclined to the Equator at an angle which grows smaller and smaller, until at the solstices it is moving parallel to the equator.

The rate of change of declination therefore also diminishes, until at the solstices it vanishes, and changes its sign.

33. Let h be the hour angle of the Star of North declination d.

h_1 „ „ South „ $2d$.

Then

$$\text{Cos. } h = \tan d \cot. l.$$

$$\text{Cos. } h_1 = \tan 2d \tan l.$$

Therefore

$$\text{Cos. } h \cos. h_1 = \tan d \tan 2d = \frac{2x^2}{1 - x^2}, \text{ if } \tan d = x.$$

But cos. $h = \dfrac{4}{5}$, cos. $h_1 = \dfrac{3}{5}$.

Therefore $\dfrac{2x^2}{1 - x^2} = \dfrac{12}{25}$.

Solving this equation we obtain tan d, and thence find l.

Ans., the declinations are 23° 45′, and 47° 30′ respectively, the latitude 28° 49′ N.

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34. *See fig.* 50.

When the ecliptic is perpendicular to the horizon, it becomes a circle of altitude, and therefore passes through the zenith.

Let AZ represent the arc of the ecliptic intercepted between the first point of Aries and the zenith.

Take QR = $1^{h.}$ $40^{m.}$ $30^{s.}$, and draw a circle of declination PR.

The point of intersection of AZ and PR gives S the place of the Sun.

In the triangle AZQ, ZQ = 15°, ZAQ = 23° 28′, ZQA = 90°.

Thence AQ may be found, and is equal to $2^{h.}$ $32^{m.}$ $27^{s.}$.

Subtracting QR, viz., $1^{h.}$ $40^{m.}$ $30^{s.}$, we obtain AR, the Sun's Right Ascension, $0^{h.}$ $51^{m.}$ $57^{s.}$.

Then in the triangle ASR, having given the side AR, the angle A, and the angle ARS a right angle, we may find SR.

Ans., 5° 34′ N.

35. *See fig.* 51.

Proceeding in the usual manner, we shall find the Ship Apparent Time to be $22^{h.}$ $58^{m.}$.

Lay off QR = $1^{h.}$ $2^{m.}$, and take RS, equal to 17°, upon the circle of declination PR.

Through S, Z describe a great circle meeting the equator in A.

Then SZA is an arc of the ecliptic, and A the first point of Aries.

In the triangle ASR we have SR = 17°, SAR = 23° 28′, and ARS = 90°.

Thus AR, the Right Ascension, may be found, and is equal to $2^{h.}$ $59^{m.}$ $4^{s.}$.

Subtracting QR, viz., $1^{h.}$ $2^{m.}$, we obtain AQ, the Sidereal Time, which is $1^{h.}$ $57^{m.}$ $4^{s.}$.

Then, in the triangle AZQ, we may find ZQ, the latitude.

Ans., 11° 59′ N.

36. *See fig.* 52.

Let O be the centre of the earth, A the position of the observer at a height AB above the earth's surface, C the target, DAC being the angle observed.

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The angle HAD is the dip of the horizon due to the height AB, which may be taken from the Tables, and is 9′ 36″.

The angle BAC = 90° − HAD − DAC = 88° 13′ 4″.

And BC = AB tan BAC, since the distance is very small.

Ans., 3083 feet.

37. *See fig.* 53.

Let AB be the lighthouse, C the foot of the cliff.

Take a point D in the horizontal line CE, such that the square on CD = the rectangle AC. CB. (Euc. II., 14).

Join AD, BD, and describe a circle about the triangle ABD.

By Euclid III., 37, CE touches the circle, and it may easily be shewn that the value of the angle ADB is greater than that subtended by the lighthouse at any other point in CE.

To find ADB, we have

$$CD^2 = 360.\ 250 = 25.\ 36.\ 100.$$

$$\therefore\ CD = 300.$$

Thus knowing AC, BC, CD we may find the angles ADC, BDC, the difference between which is the angle ADB.

Ans., 10° 23′.

38. *See fig.* 54.

Let h_1 be the distance in hour angle of the Moon from the Meridian at rising.

Let h_2 be the hour angle of the Sun at setting, R_1PR_2 be the elapsed time from the setting of the Sun to the rising of the Moon, so that the Sun was upon the circle of declination PR_2 when the Moon rose.

Then we have from the figure—

$$h_1 + h_2 + R_1R_2 = AR_3 + R_2A.$$

$\therefore\ h_1 + h_2 +$ elapsed time = R.A. of Moon + $24^{h.}$ − R.A. Sun.

Whence $h_1 + h_2 = 12^{h.}\ 24^{m.}\ 44{\cdot}6^{s.}$.

And as in previous problems we may obtain the relation—

$$\text{Tan}\ \frac{h_1 - h_2}{2} = -\frac{\text{sin.}\ \overline{d_1 - d_2}}{\text{sin.}\ \overline{d_1 + d_2}}\ \text{cot.}\ \frac{h_1 + h_2}{2},$$ where d_1, d_2 are the declinations of the Moon and Sun respectively.

Ans., 41° 44′ N.

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39. If h_1, h_2 be the distances in hour angle from the meridian of Procyon and α Hydræ respectively, when each was in the horizon, by a suitable diagram we shall find that—

Elapsed time (in sidereal time) = difference of hour angles + difference of right ascension.

Whence $h_1 - h_2 = 1^{h.}\ 6^{m.}\ 34^{s.}$.

We can then proceed as in former examples.

Ans., 50° 23′ N.

40. *See fig.* 55.

Let M_1 be the position of the Moon when on the prime vertical East of Meridian, M_2 be her position when on the prime vertical West of Meridian, and let h_1, h_2 be the distances in hour angle East and West of Meridian respectively, p_1, p_2 being the polar distances; and let R_1R_2 be the elapsed sidereal time between the two passages over the prime vertical.

To find p_1, p_2, if we assume that the Moon was on the meridian $4^{h.}\ 35^{m.}$ after passing the prime vertical, we shall have—

p_1 = 73° + change of declination for $4^{h.}\ 35^{m.}$ = 73° 29′ 47″.

p_2 = 73° − change of declination for $4^{h.}\ 35^{m.}$ = 72° 30′ 13″.

Next to find $h_1 + h_2$.

We have from the figure—

$h_1 + h_2 = R_2R_1 - R_2R_2$.

= elapsed sidereal time − increase of Moon's R.A.

$= 9^{h.}\ 11^{m.}\ 30{\cdot}3^{s.} - 22^{m.} = 8^{h.}\ 49^{m.}\ 30{\cdot}3^{s.}$.

Having then $h_1 + h_2$, we may determine h_1, h_2 by means of the equations—

$$\text{Cos. } h_1 = \text{cot. } p_1 \text{ cot. } l.$$

$$\text{Cos. } h_2 = \text{cot. } p_2 \text{ cot. } l.$$

And thence the latitude.

Ans., 37° 9′ N.

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